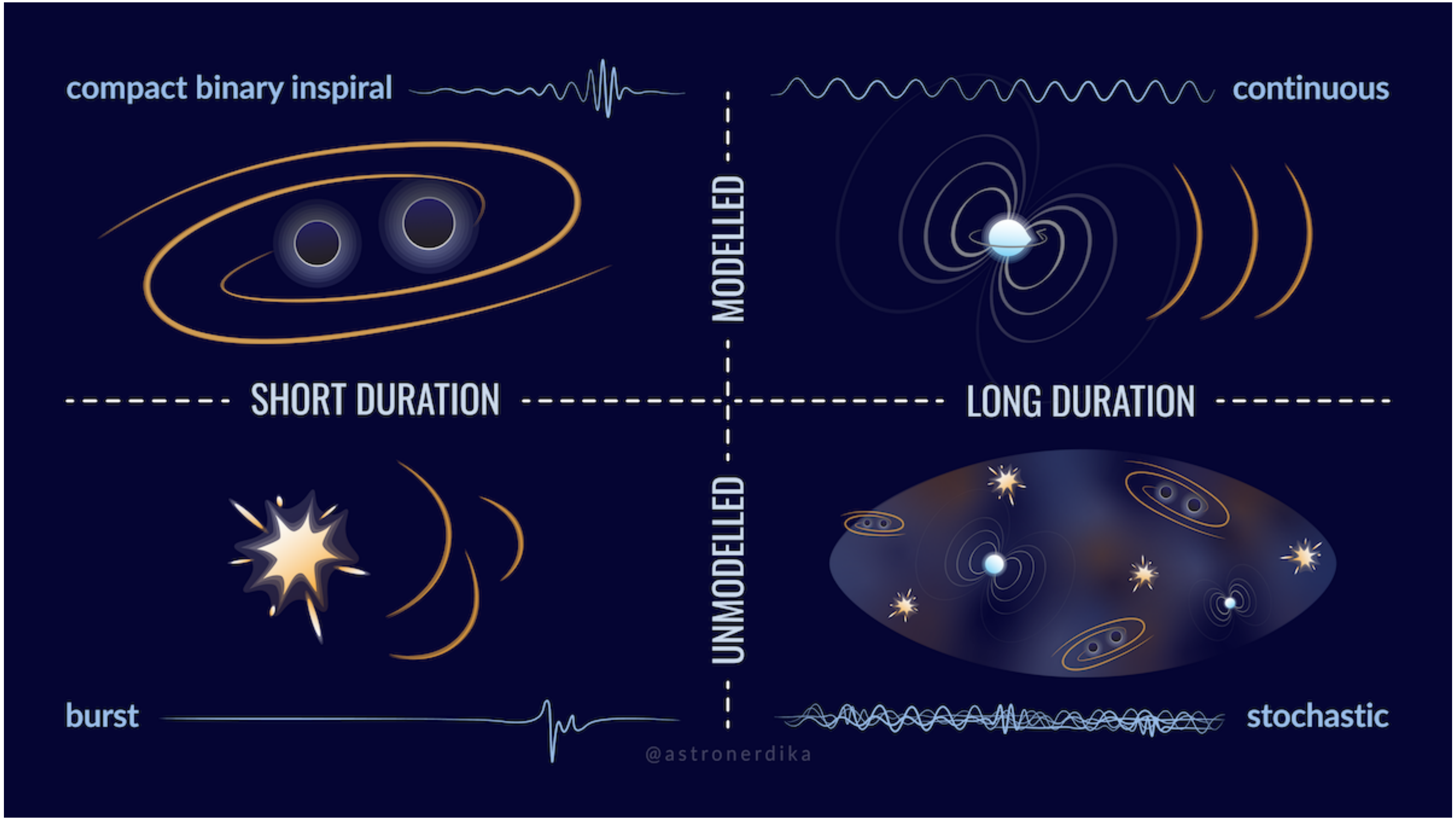
Stochastic GW background

Chia-Hsuan Hsiung on behave of stochastic isotropic subgroup

Content

- What is stochastic GW background
- How to detect?

What is stochastic GW background ?



What is stochastic GW background ?

Superposition of many GW signals which are

uncorrected

Fractional GW energy density $$\Omega_{GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{GW}}{dlnf} = \frac{32\pi^3}{3H_0^2} f^3 I(f)$$

GW intensity $$I(f) = \frac{1}{2}\sum_A |h_A(f)|^2$$

Critical energy density ρ_c

Hubble constant H_0

What is stochastic GW background ?

From the Friedman equation we have:

$$H(z) = H_0(\Omega_R(1+z)^4 + \Omega_M(1+z)^3 + \Omega_\kappa(1+z)^2 + \Omega_\Lambda)^{\frac{1}{2}}$$

$$\Omega_R = \Omega_\gamma + \Omega_N + \Omega_{GW} + \ldots\ldots$$

Important constraints on Ω_{GW}

$$\Omega_{\rm GW} < 1.2 \times 10^{-6} \left(\frac{H_0}{100 \text{ km s}^{-1} \text{ Mpc}^{-1}} \right)^{-2}$$

What is stochastic GW background ?

$$\Omega_{GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{GW}}{dlnf} = \frac{32\pi^3}{3H_0^2} f^3 I(f)$$

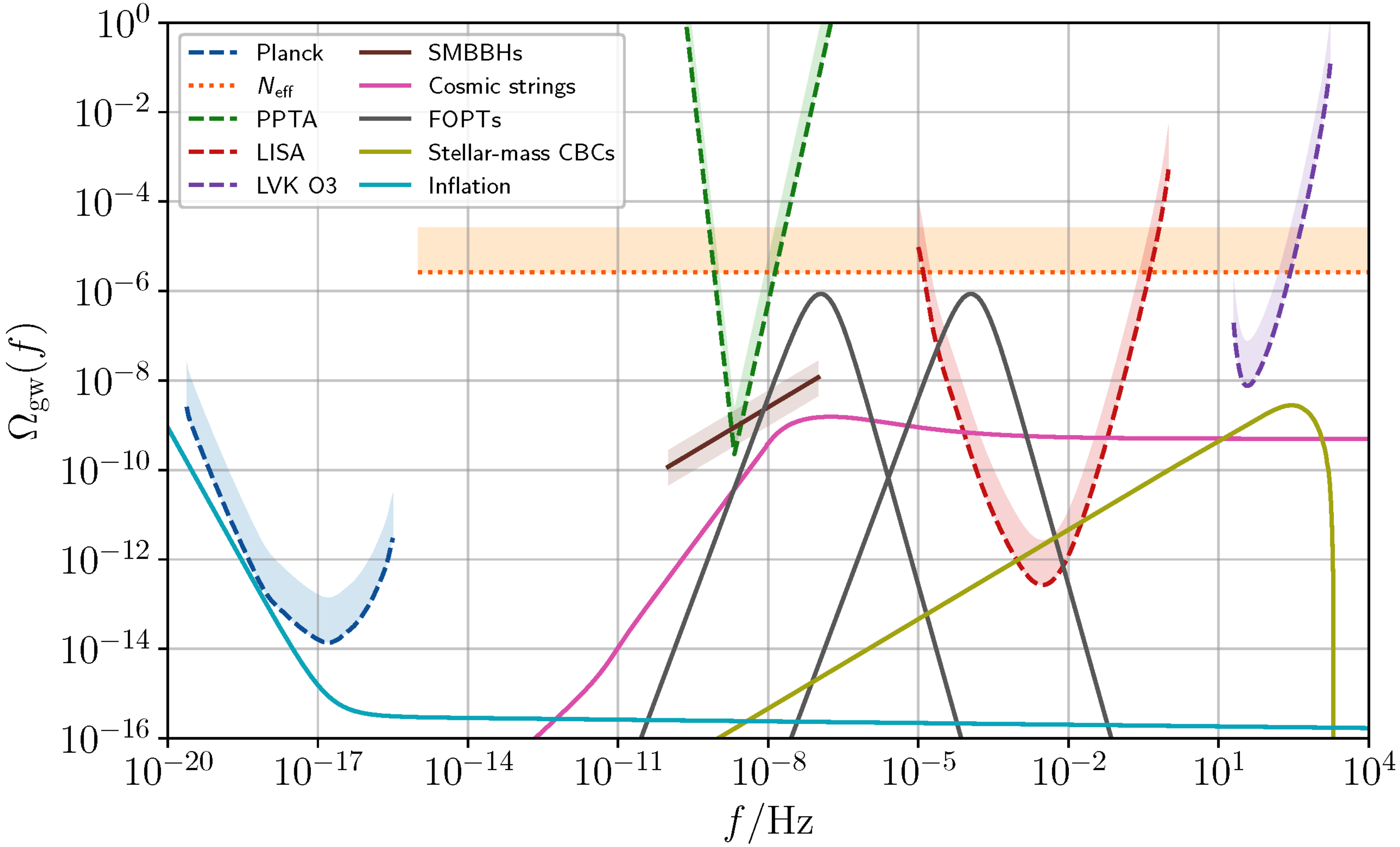
Superposition of many GW signals which are uncorrected

Cosmological:

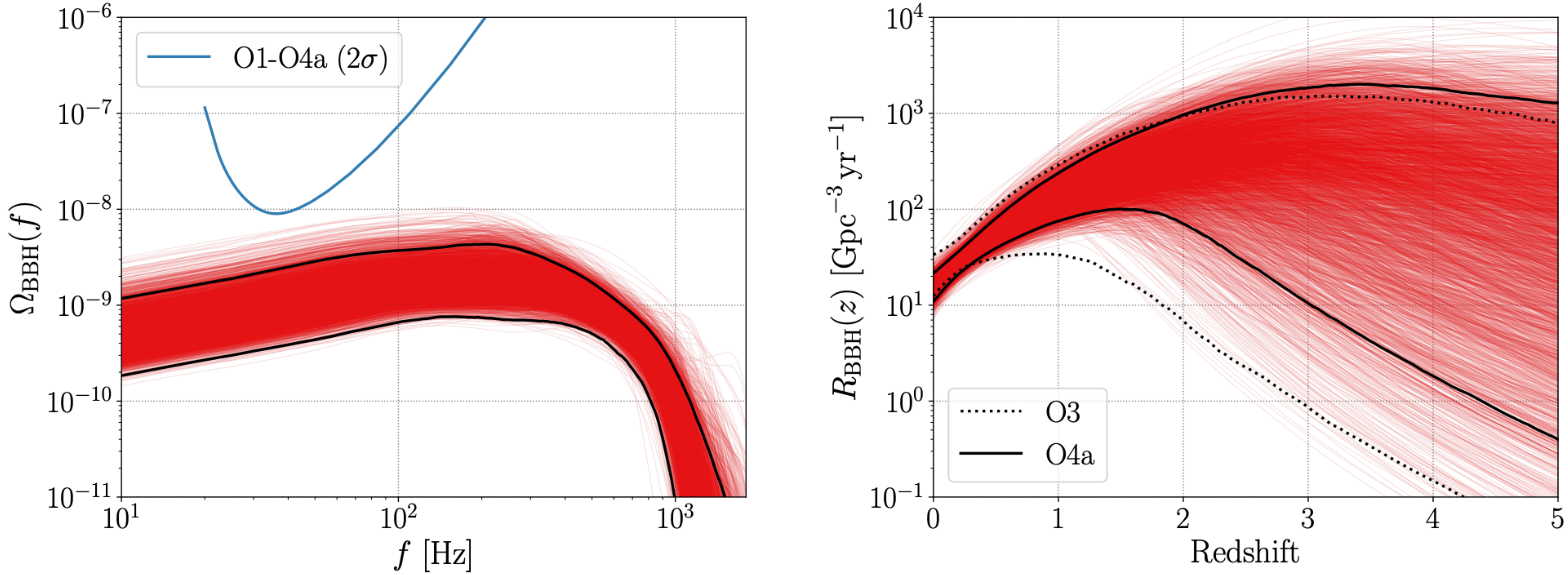
- Cosmic strings
- Inflation
- Phase transitions and others

Astrophysical:

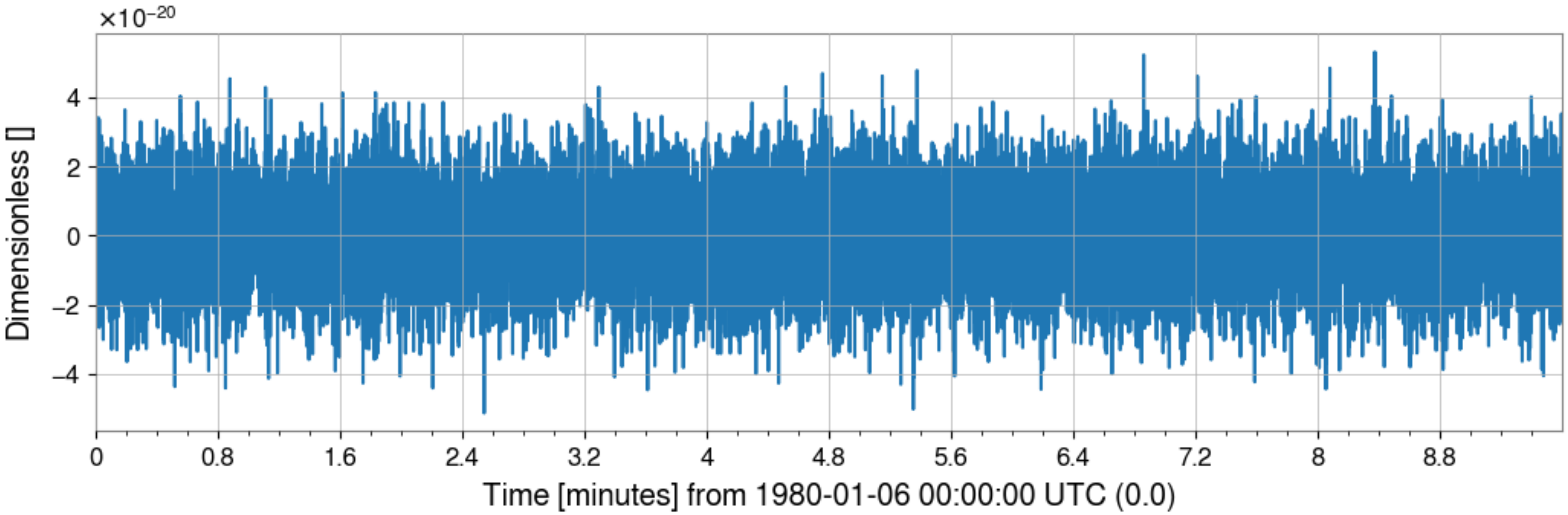
- BBH, NSBH, SMBBHs…
- Supernove
- Rotating NS and others



What is stochastic GW background ?



How to detect?



Main detection strategy in pygwb : Cross correlation

Cross-correlation of GW

Uncorrelated, Gaussian, unpolarized

$$s_I(t) = h_I(t) + n_I(t)$$

- It's hard to claim that the signal detected with one detector. However, we can confirm the gravitational wave once we have another detector because signal will have a time shift in two detectors.

$$\langle \tilde{s}_{I_1}(f)\tilde{s}^*_{I_2}(f)\rangle = \langle \tilde{h}_{I_1}(f)\tilde{h}^*_{I_2}(f)\rangle + \langle \tilde{h}_{I_1}(f)\tilde{n}^*_{I_2}(f)\rangle$$
$$+\langle \tilde{n}_{I_1}(f)\tilde{h}^*_{I_2}(f)\rangle + \langle \tilde{n}_{I_1}(f)\tilde{n}^*_{I_2}(f)\rangle$$

- Because the signal and noise are uncorrelated, and the expectation value of the noise is zero in principle, which simply the equation into

$$\langle \tilde{s}_{I_1}(f)\tilde{s}^*_{I_2}(f)\rangle \approx \langle \tilde{h}_{I_1}(f)\tilde{h}^*_{I_2}(f)\rangle$$

which can help us draw out the GW sneaking inside the noise.

How to detect?

What about the detector site?

From antenna pattern to overlap reduction function

- In gravitataional wave radiometry, we first set up the notation we need. In terms of spherical polar coordinates (θ, ϕ), the source direction is given by

$$\hat{\Omega} = cos\phi\ sin\theta\ \hat{x} + sin\phi\ sin\theta\ \hat{y} + cos\theta\ \hat{z}$$

and the polarization tensor $e^A(\hat{\Omega})$ are defined by the following expression:

$$e^{\times}(\hat{\Omega}) = \hat{e}_\theta \otimes \hat{e}_\theta - \hat{e}_\phi \otimes \hat{e}_\phi \quad , \quad e^{+}(\hat{\Omega}) = \hat{e}_\phi \otimes \hat{e}_\theta + \hat{e}_\phi \otimes \hat{e}_\theta$$

$$\hat{e}_\theta = cos\phi\ cos\theta\ \hat{x} + sin\phi\ cos\theta\ \hat{y} - sin\theta\ \hat{z}$$

$$\hat{e}_\phi = -\ sin\phi\ \hat{x} + cos\phi\ \hat{y}$$

and $(\hat{e}_\theta, \hat{e}_\phi)$ are two orthonormal basis vectors on a two-sphere.(10)

From antenna pattern to overlap reduction function

- Now we consider two GW detectors located at $x_I(t)$, $I = 1{,}2$. The detector frames are denoted by the coordinates (X_I, Y_I, Z_I) where the arms of the respective detectors lie along the (X_I, Y_I) axis. Then the detector tensors d_I are given by

$$d_I = \frac{1}{2}[\hat{X}_I \otimes \hat{X}_I - \hat{Y}_I \otimes \hat{Y}_I]$$

- Owing to the Earth's rotation, the detector coordinates and d_I are functions of t. Thus in matrix form:

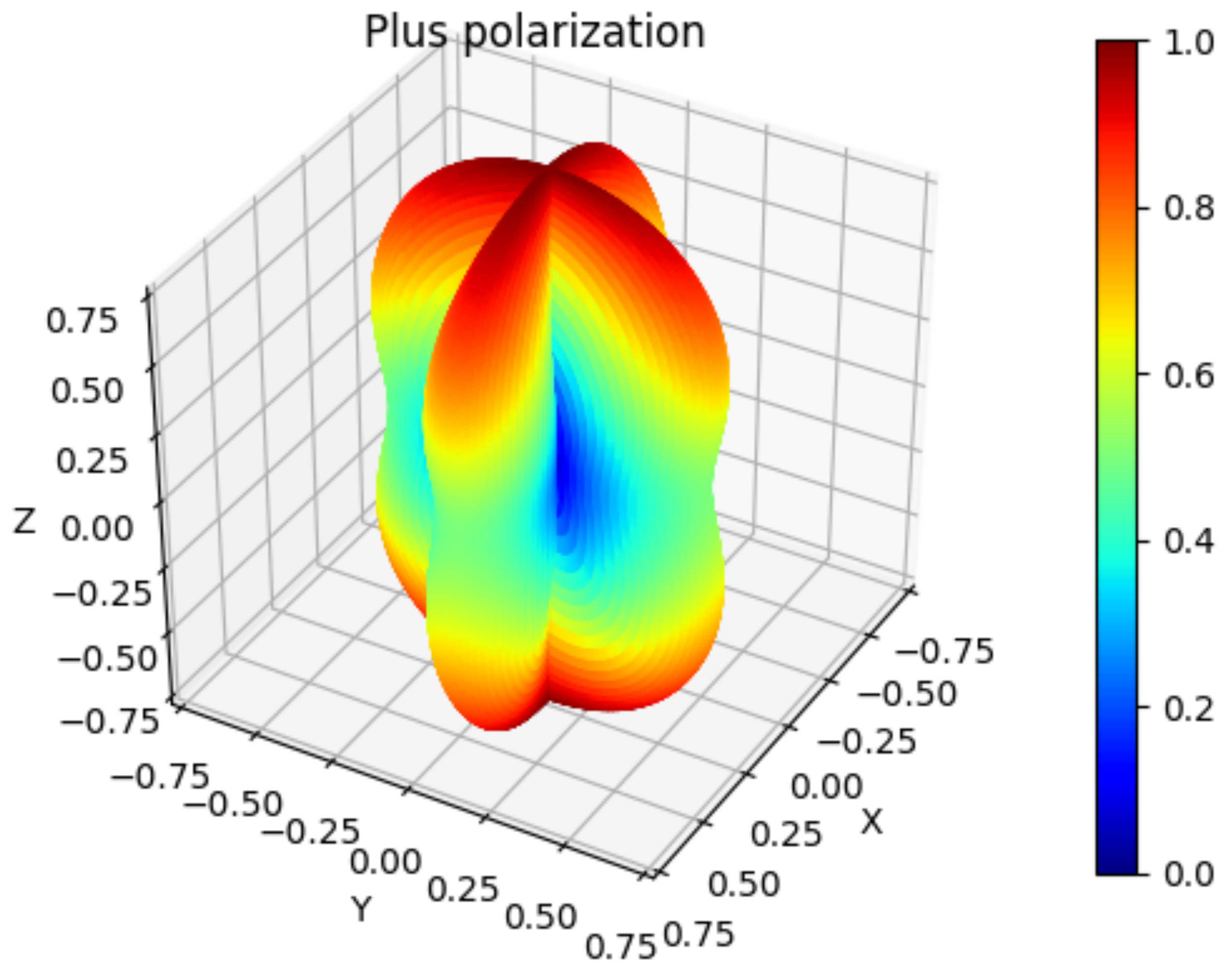
$$d_I(t) = R^T(t) \cdot d_I(t=0) \cdot R(t)$$

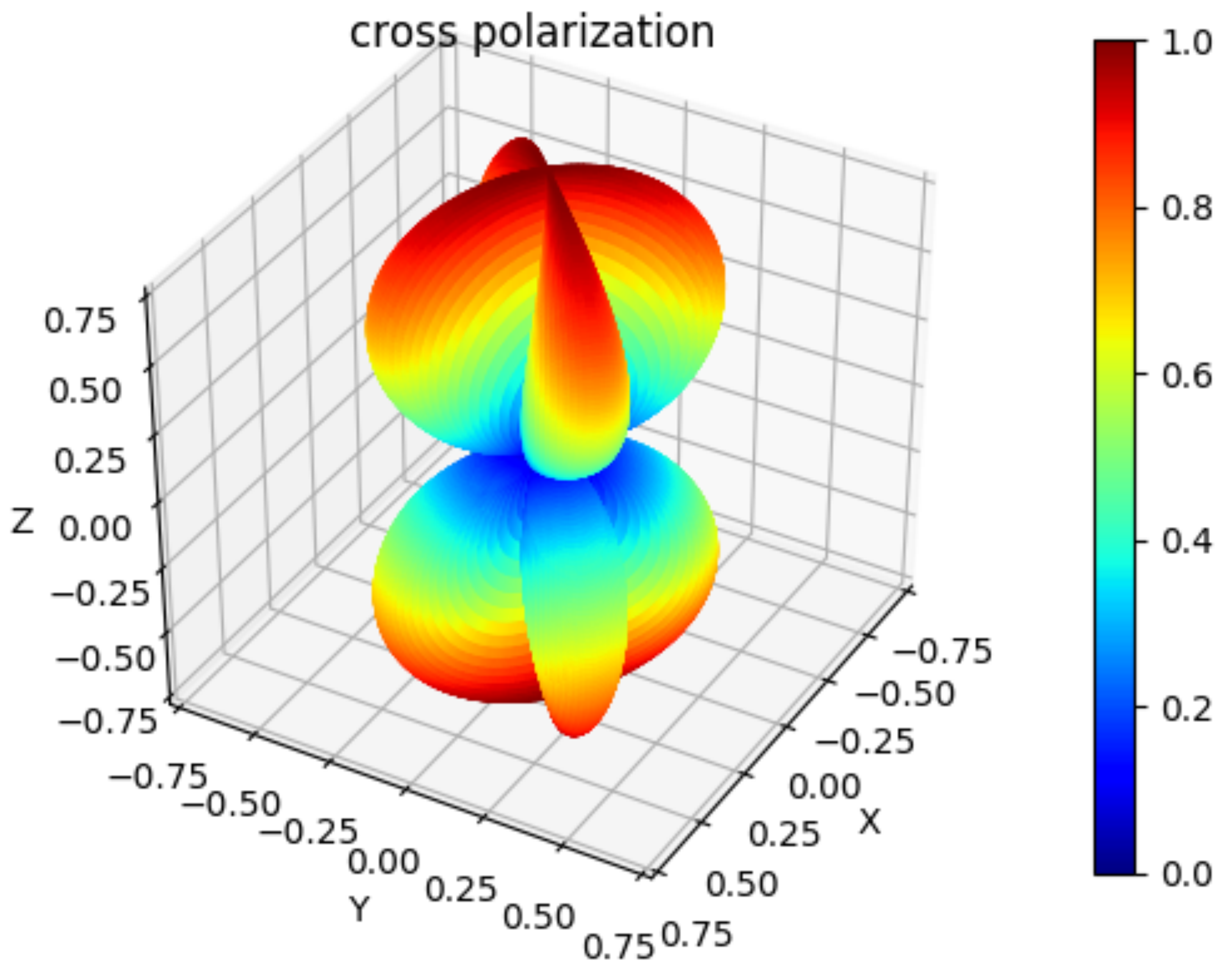
where $R(t)$ is the rotation matrix describing a rotation of angle $\omega_E t$ around the Earth's rotation axis, namely, the z-axis. Now we can define the antenna pattern functions

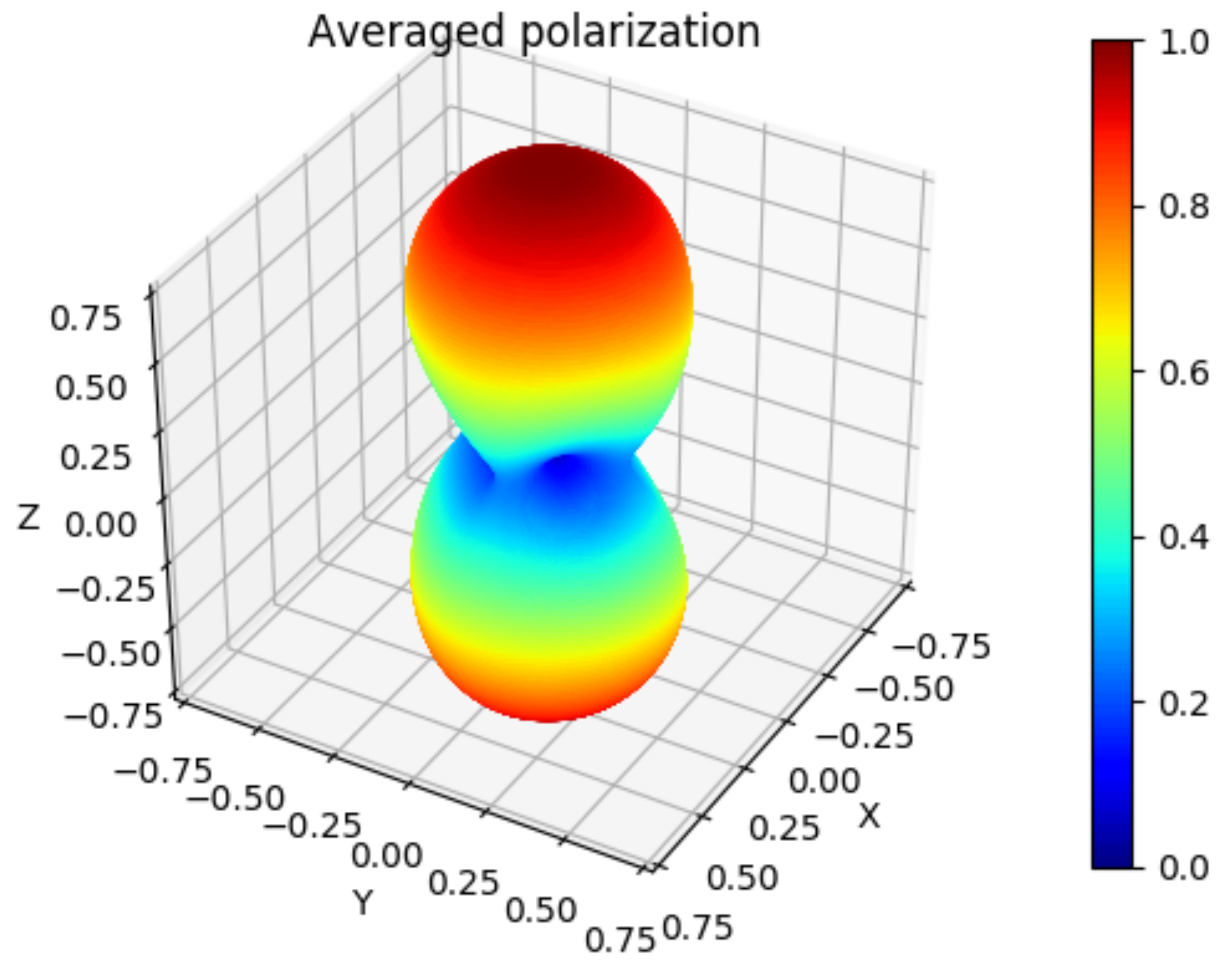
$$F_I^A(t, \hat{\Omega}) = \varrho_{ij}^A(\hat{\Omega}) d_I^{ij}(t)$$

for a wave that incident from the direction $\hat{\Omega}$.

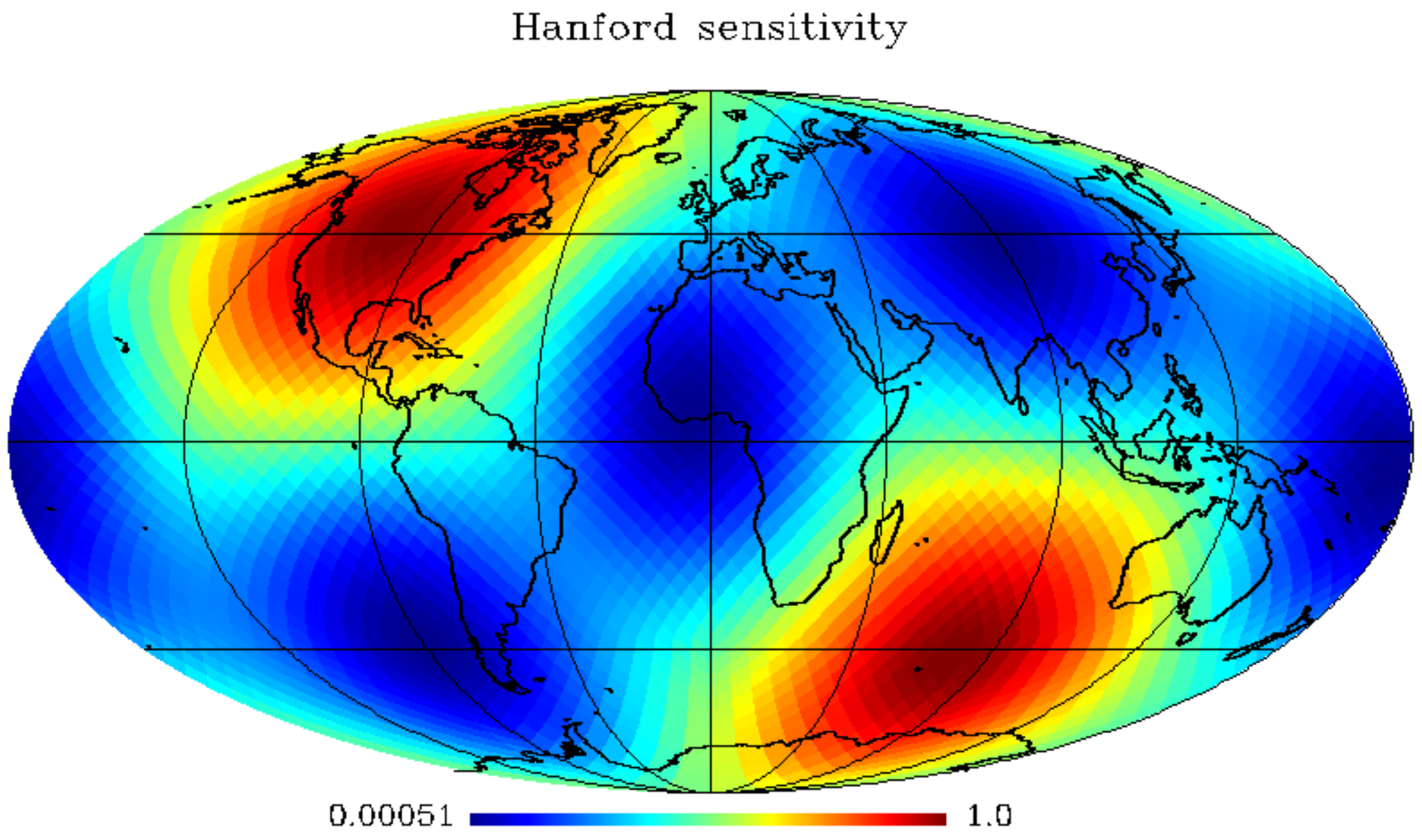
From antenna pattern to overlap reduction function

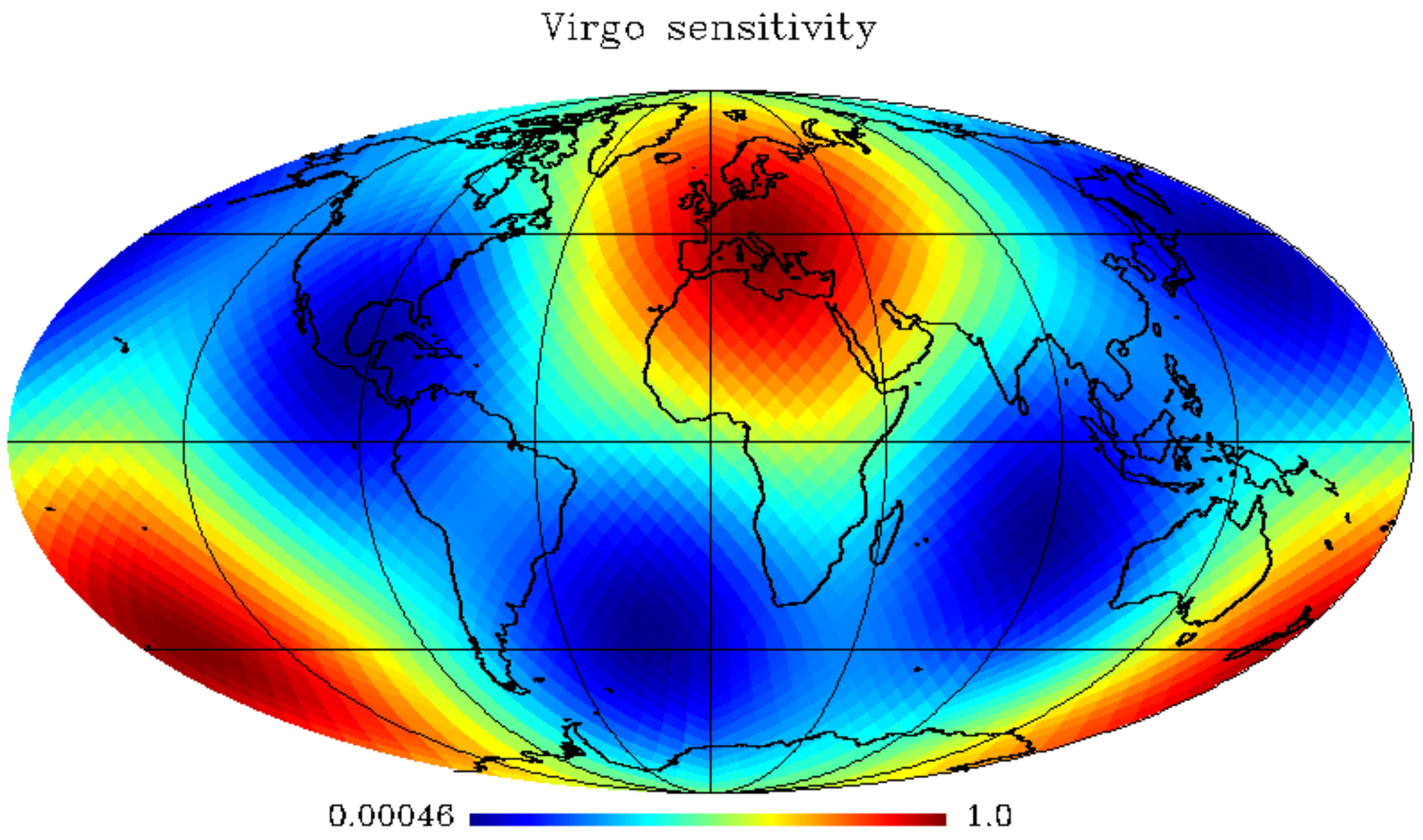






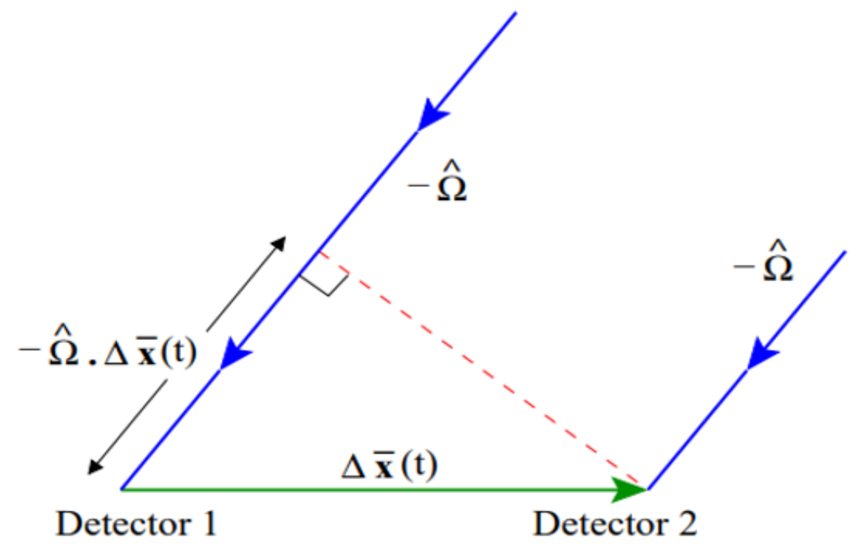
$$F_I^A(t, \hat{\Omega}) = \varrho_{ij}^A(\hat{\Omega}) d_I^{ij}(t)$$


Hanford sensitivity
0.00051
1.0

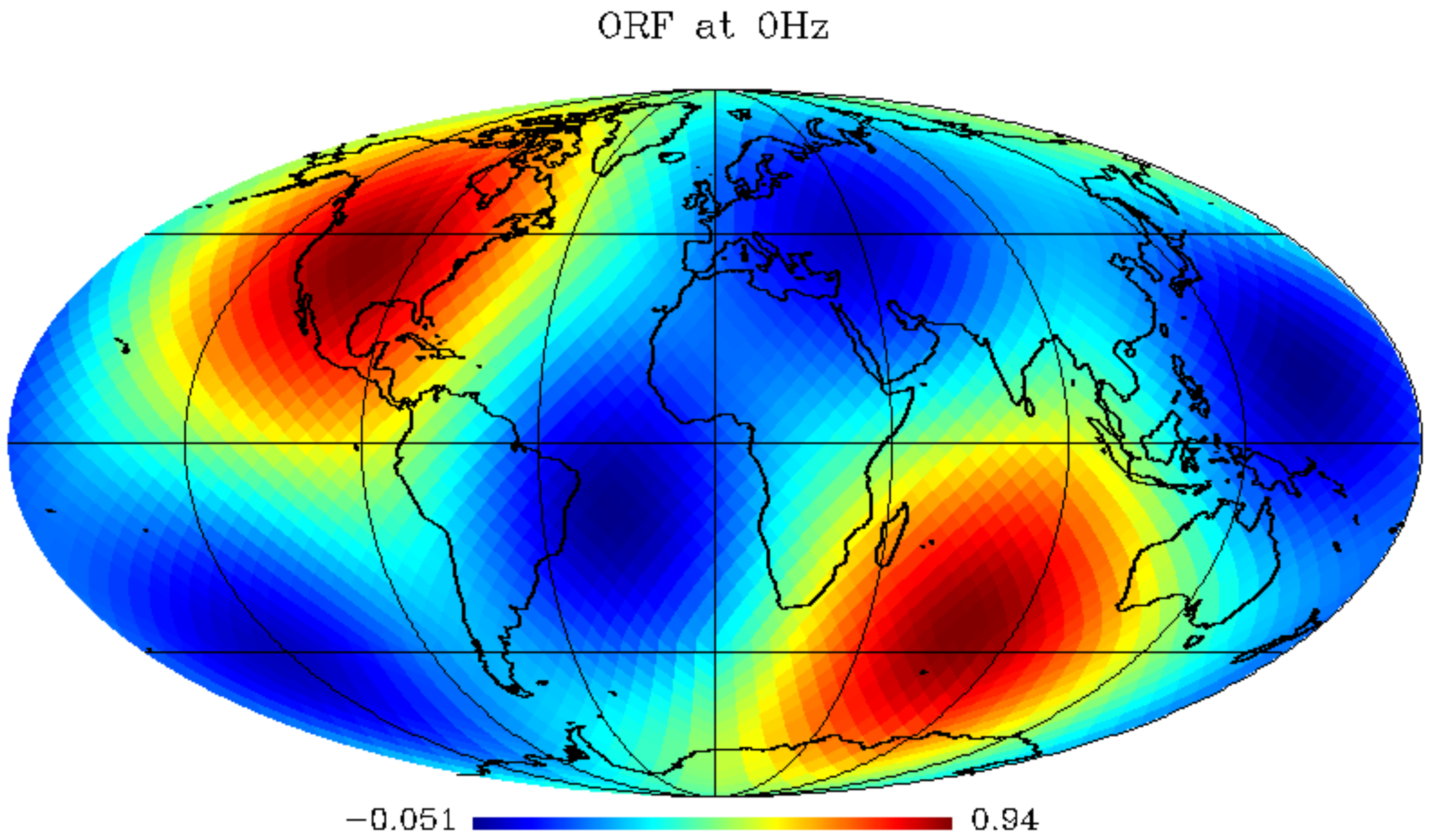

Virgo sensitivity
0.00046
1.0

From antenna pattern to overlap reduction function

- Finally, consider the rotation of the Earth and time delay from one to the other detector, we can generate a geometric factor γ^I known as the overlap reduction function (ORF),



where $\Delta x_I(t)$ is the



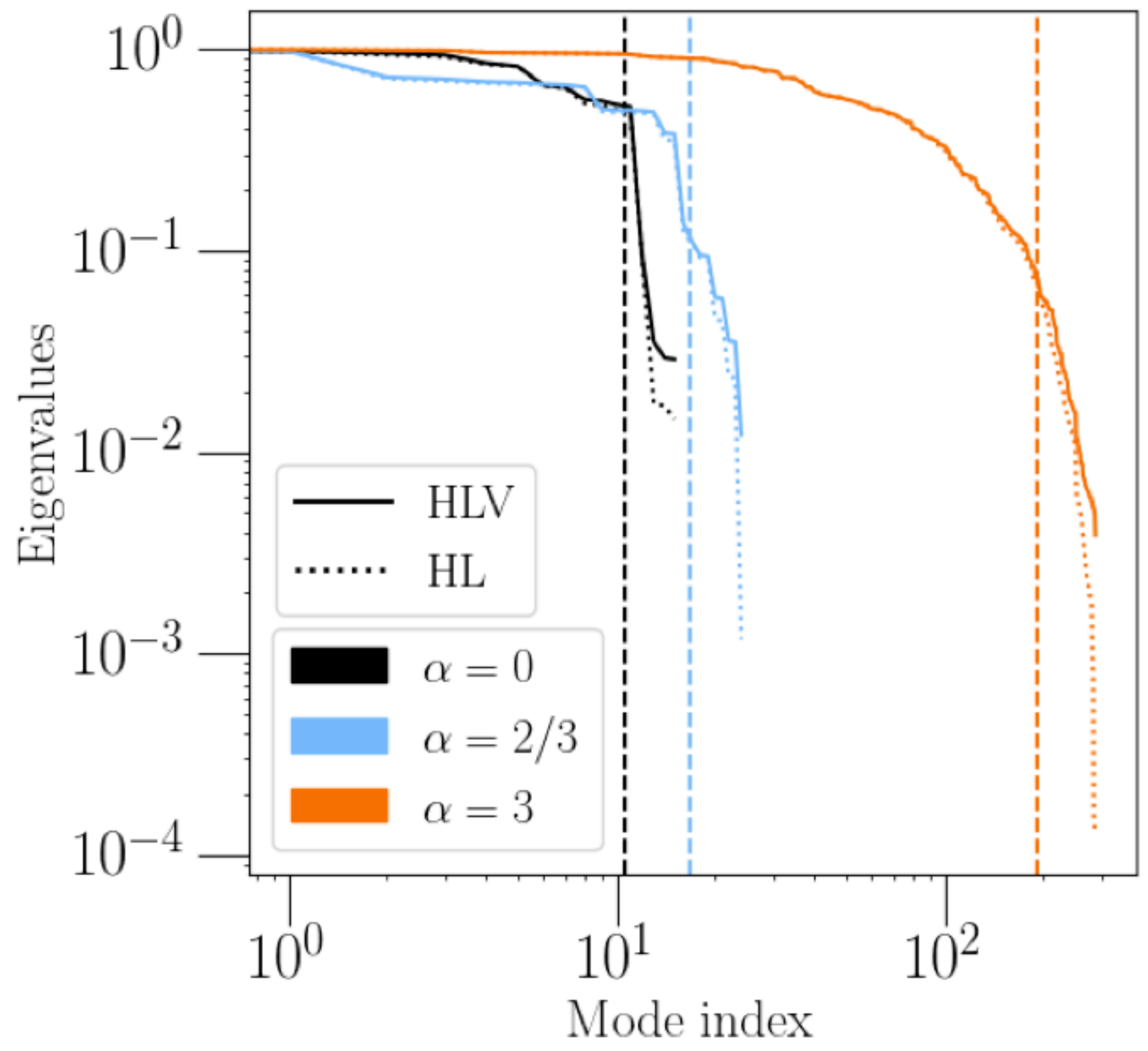
, I denotes the baseline constituted by two

The limit of overlap reduction function

$$\mathbf{X} = \frac{4}{\tau}\sum_{Ift}\frac{H(f)\gamma^{I*}_{ft,p}}{P_{\mathcal{I}_1}(t;f)P_{\mathcal{I}_2}(t;f)}\tilde{s}^*_{\mathcal{I}_1}(t;f)\tilde{s}_{\mathcal{I}_2}(t;f) \quad \mathbf{\Gamma} = 4\sum_{Ift}\frac{H^2(f)}{P_{\mathcal{I}_1}(t;f)P_{\mathcal{I}_2}(t;f)}\gamma^{I*}_{ft,p}\gamma^{I}_{ft,p'}$$

$$\hat{\mathcal{P}}_p \equiv \hat{\mathcal{P}} = \mathbf{\Gamma}^{-1}\cdot\mathbf{X},$$

Singular Value Decomposition helps with the inversing matrix problem



How to detect?

Recall the equations before, and also considering the simplest case :

Unpolarized, Gaussian, isotropic

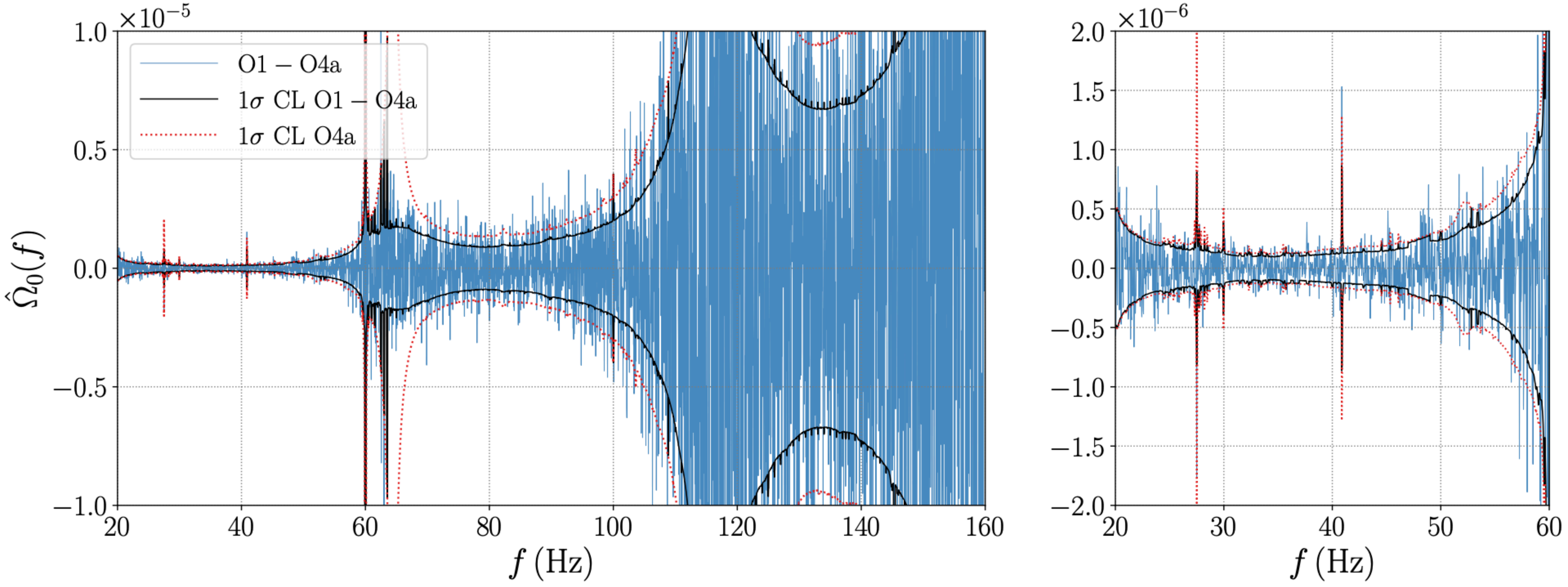
then we can generate the cross-correlation statistic for GWB search

$$\langle C_{12}(f)\rangle = R_1(f)R_2^*(f)\langle \tilde{h}_{I_1}(f)\tilde{h}_{I_2}^*(f)\rangle = T_{OBS}\Gamma^I(f)I(f)$$

where

$$\langle C_{12}(f)\rangle \approx T_{OBS}\Gamma^I(f)f^{-3}\Omega_{GW}(f)$$

How to detect?

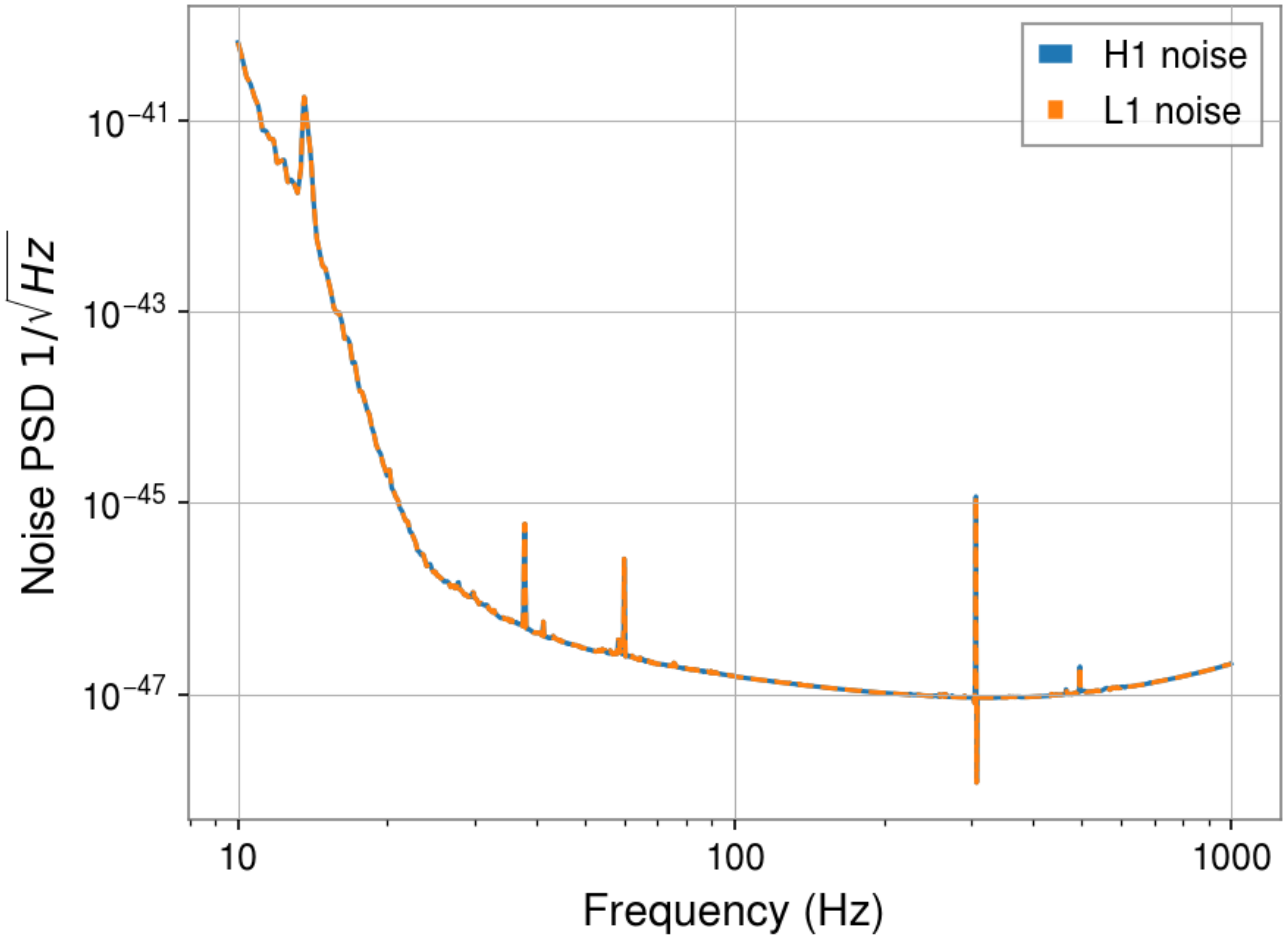


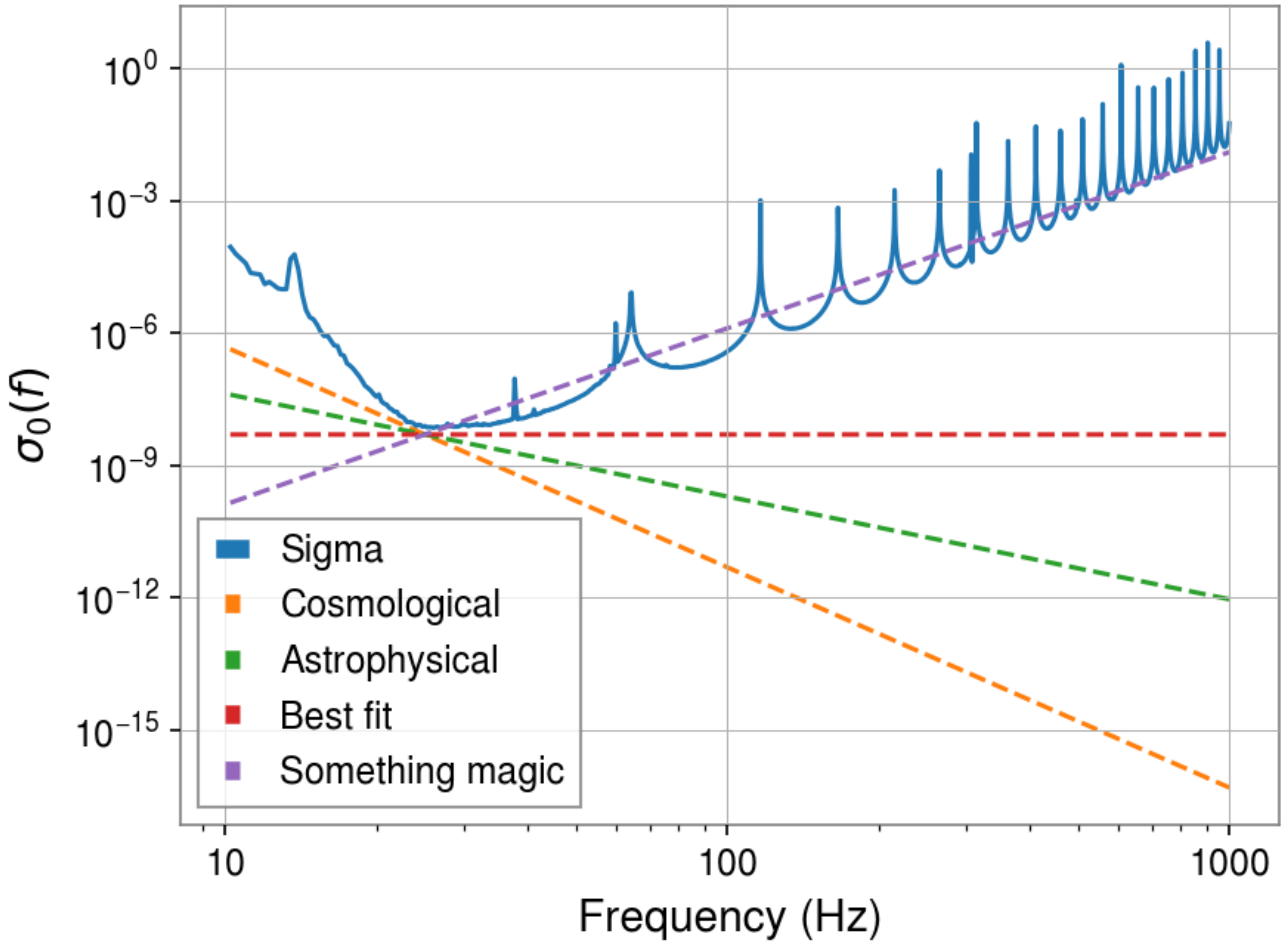
How to detect?

$$\Omega_{GW} \propto f^{\alpha}$$

Fix the alpha

$$\sigma_0^2(f) = \frac{1}{2T\Delta f}\frac{P_I(f)P_J(f)}{\gamma_{IJ}^2(f)S_0^2(f)}$$

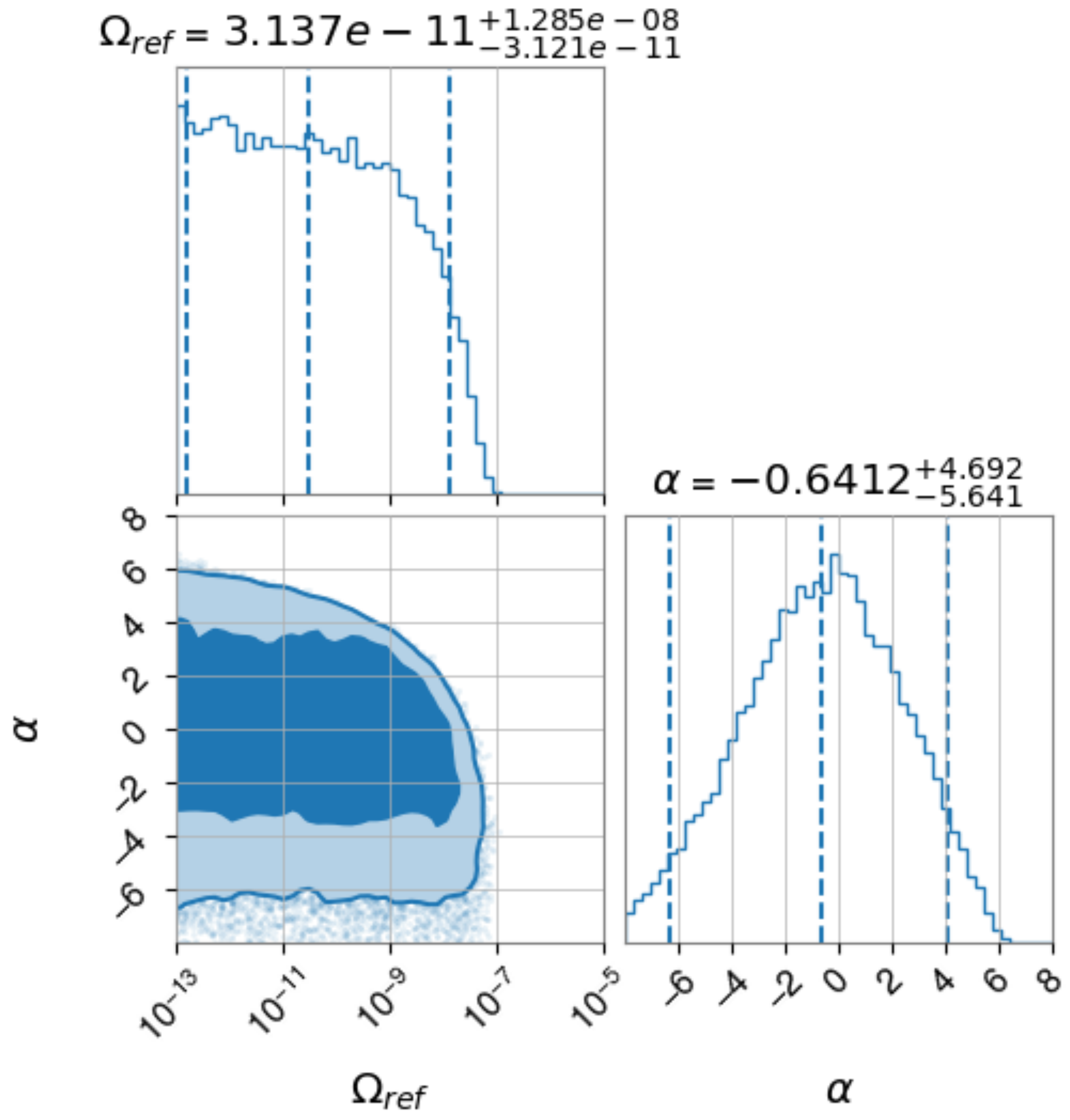


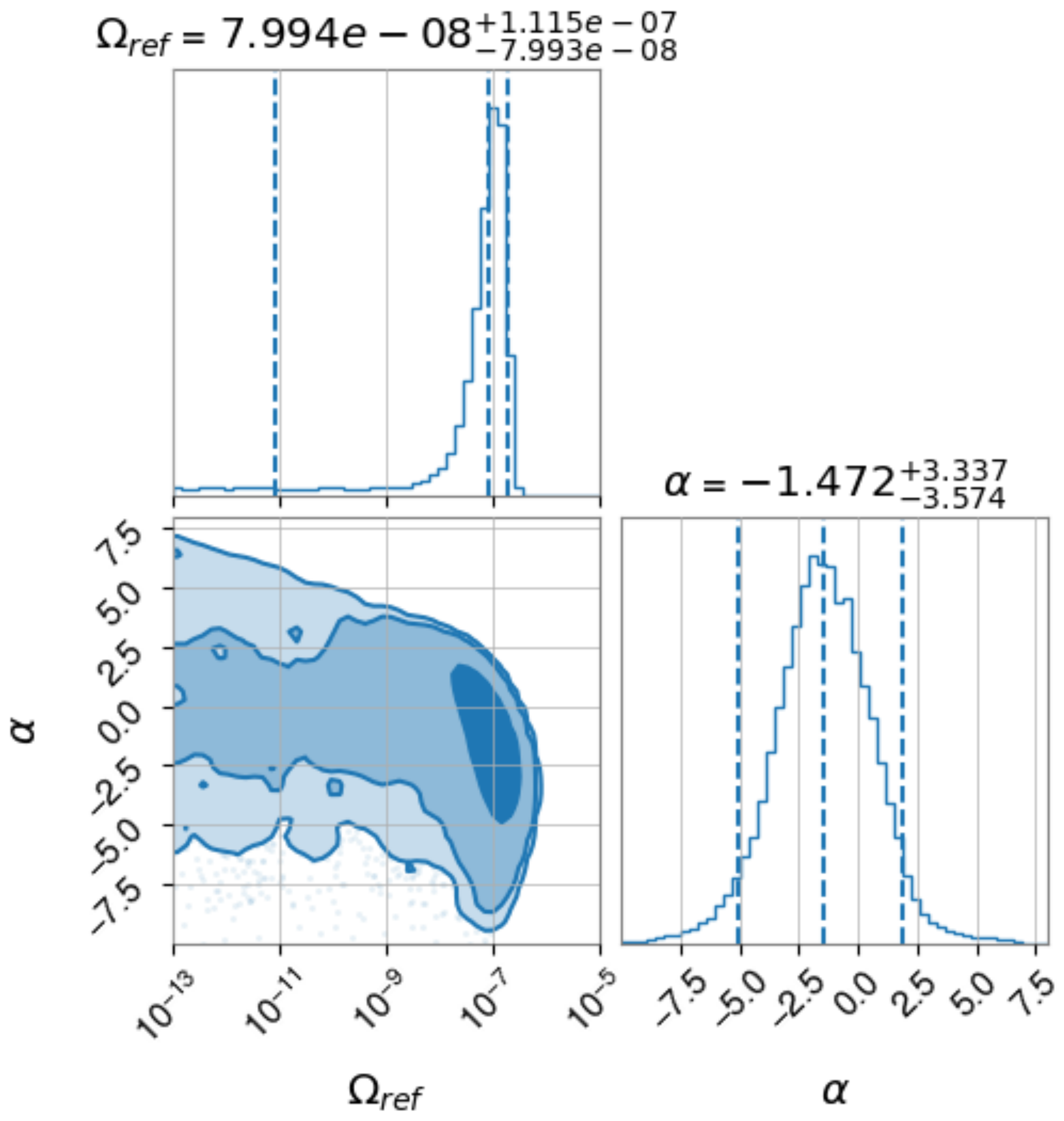


How to detect?

$$\Omega_{GW} \propto f^{\alpha}$$

Parameter estimation





How to detect?

Bayesian fit estimate

$$p(\hat{\Omega}_0|\Theta,\Lambda) \propto \exp\left[-\frac{1}{2}\sum_f \frac{(\hat{\Omega}_0(f) - \Lambda\Omega_{\rm GW}(f;\Theta))^2}{\sigma_0^2(f)}\right]$$

Calibration errors Λ

GW model $\Omega_{GW}(f;\theta)$

Model selection : Bayes factor

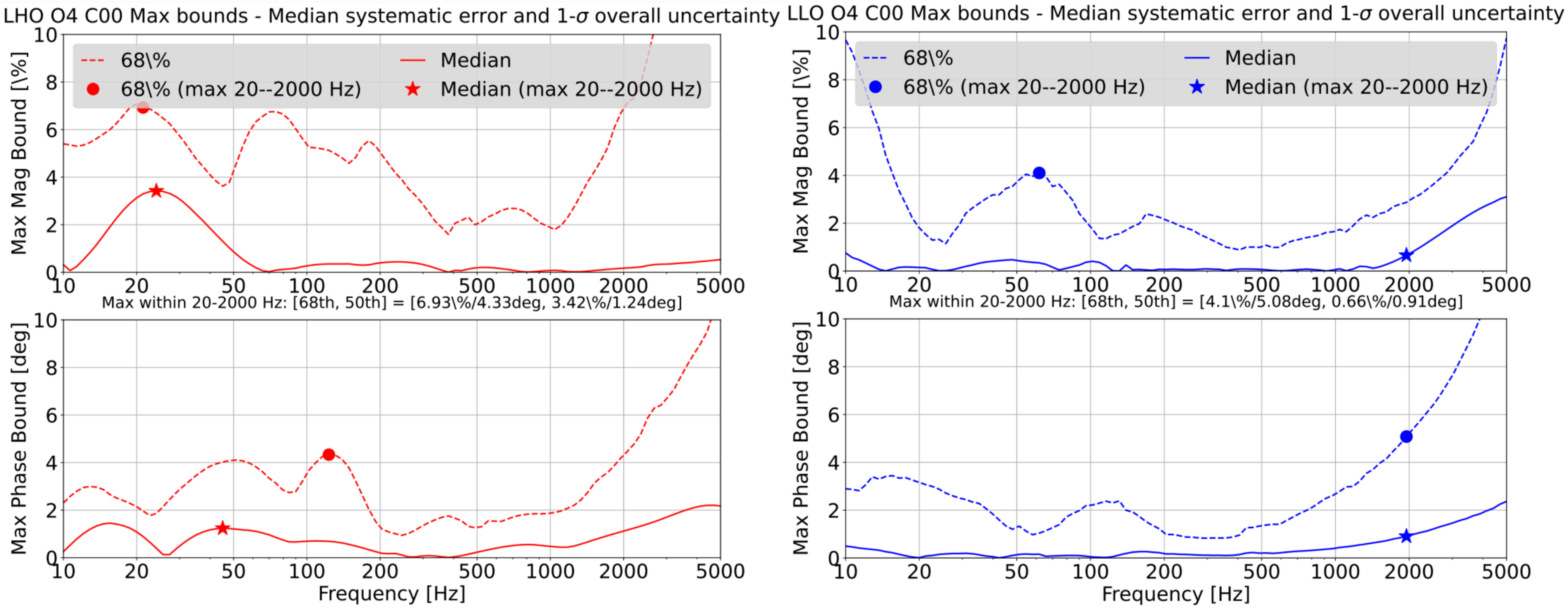
$$\kappa = \frac{\hat{p}(M_1; data)}{\hat{p}(M_2; data)}$$

Noise by default

How to detect?

$$p(\hat{\Omega}_0|\Theta,\Lambda) \propto \exp\left[-\frac{1}{2}\sum_f \frac{(\hat{\Omega}_0(f) - \Lambda\Omega_{\rm GW}(f;\Theta))^2}{\sigma_0^2(f)}\right]$$

Calibration errors Λ



How to detect?

$$p(\hat{\Omega}_0|\Theta,\Lambda) \propto \exp\left[-\frac{1}{2}\sum_f \frac{(\hat{\Omega}_0(f) - \Lambda\Omega_{\rm GW}(f;\Theta))^2}{\sigma_0^2(f)}\right]$$

Calibration errors Λ

	LHO (%)	LLO (%)
Max 68% (magnitude)	6.93	4.10
Max 68% (phase)	4.33	5.08
Max 50% (magnitude)	3.42	0.66
Max 50% (phase)	1.24	0.91

- We only keep the maximum value of the magnitude at 68%
- Maximum values of phase is ~ or < 5 deg → imaginary part is negligible
- As expected, uncertainties at LLO are smaller

How to detect?

$$p(\hat{\Omega}_0|\Theta, \Lambda) \propto \exp\left[-\frac{1}{2}\sum_f \frac{(\hat{\Omega}_0(f) - \Lambda\Omega_{\rm GW}(f;\Theta))^2}{\sigma_0^2(f)}\right]$$

GW model

Noise

$$\Omega_{GW}(f;\theta) = 0$$

Simple power law

$$\Omega_{GW}(f;\theta) = \Omega_{ref}(\frac{f}{25Hz})^{\alpha}$$

Magnetics

$$\Omega_{GW}(f;\theta) = \Omega_{ref}(\frac{f}{25Hz})^{\alpha} + \Omega_{MAG}(f;MAG)$$

Scalar-vector-tensor (SVT) power law

$$\Omega_{GW}(f;\theta) = \sum_p \frac{\gamma^p_{IJ(f)}}{\gamma_{IJ}(f)}\Omega^{(p)}_{ref}(\frac{f}{25Hz})^{\alpha}$$

CBC

$$\Omega_{\rm GW}(f;\theta_k) = \frac{f}{\rho_c H_0}\int_0^{z_{\max}} dz \frac{R_m(z;\theta_k)\langle\frac{dE_{\rm GW}}{df_s}(f_s;\theta_k)\rangle}{(1+z)E(\Omega_{\rm m,0},\Omega_{\Lambda,0},z)}$$

How to detect?

$$p(\hat{\Omega}_0|\Theta,\Lambda) \propto \exp\left[-\frac{1}{2}\sum_f \frac{(\hat{\Omega}_0(f) - \Lambda\Omega_{\rm GW}(f;\Theta))^2}{\sigma_0^2(f)}\right]$$

	Uniform prior			**Log-uniform prior**		
α	O1-O4a	O1-O3	Improvement	O1-O4a	O1-O3	Improvement
0	8.6×10^{-9}	1.7×10^{-8}	2.0	2.8×10^{-9}	5.8×10^{-9}	2.1
2/3	6.3×10^{-9}	1.2×10^{-8}	1.9	2.0×10^{-9}	3.4×10^{-9}	1.7
3	1.0×10^{-9}	1.3×10^{-9}	1.3	3.2×10^{-10}	3.9×10^{-10}	1.2
Marginalized	1.5×10^{-8}	2.7×10^{-8}	1.8	2.9×10^{-9}	6.6×10^{-9}	2.3

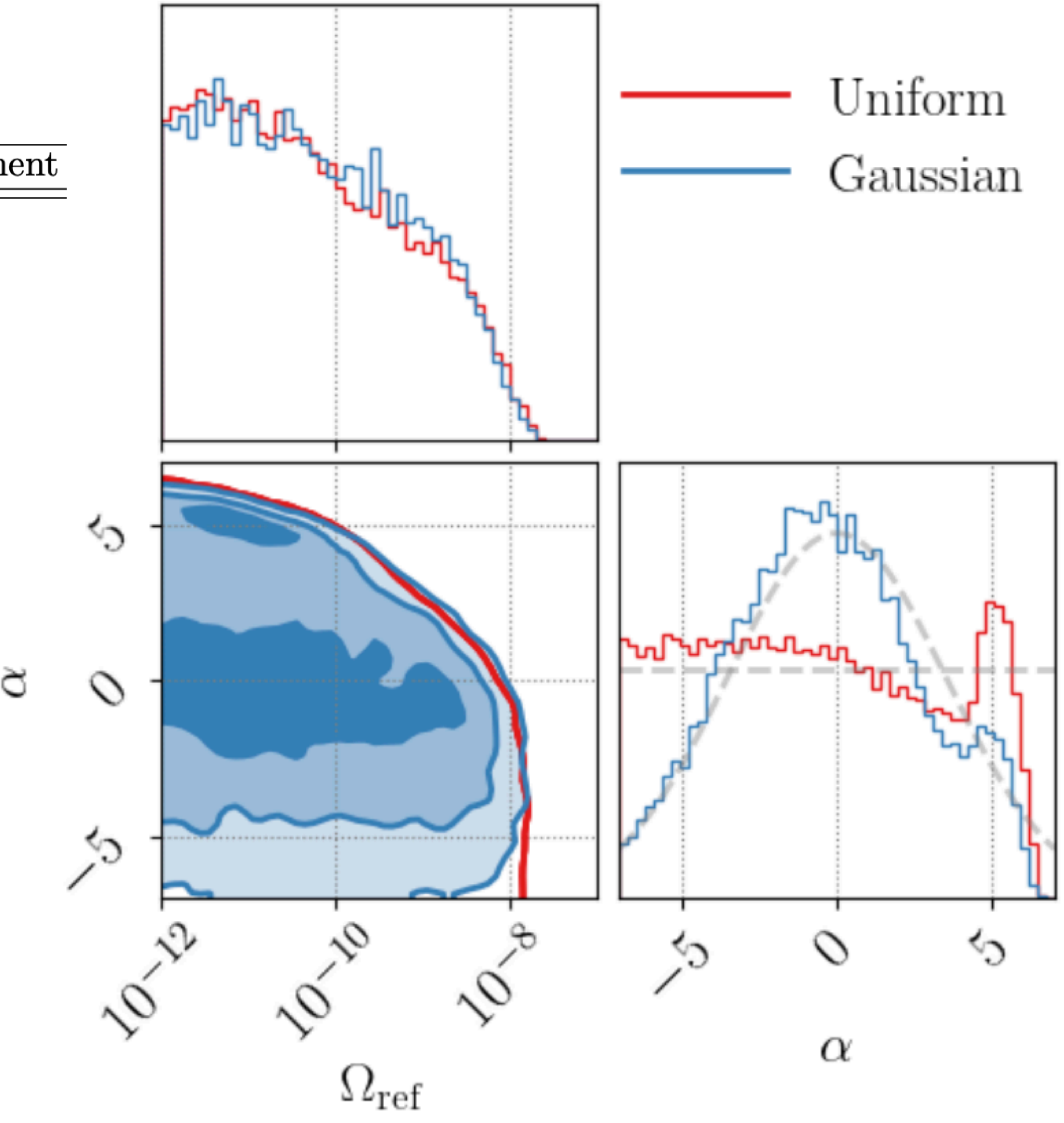
```
kwargs_PL = {"baselines": [HL_O4c1],"model_name": "PL","fref": fref}

model_PL = pe.PowerLawModel(**kwargs_PL)

priors= {'omega_ref': bilby.core.prior.LogUniform(1e-13, 1e-6, '$\\Omega_{\\rm ref}$'),

        "alpha":bilby.core.prior.Uniform(-3.5, 3.5,'$\\alpha$')}


hlv_pl=bilby.run_sampler(likelihood=model_PL,priors=priors,sampler="dynesty",
                         nlive=5000, walks=30, npool=5,printdt=500, maxmcmc=10000,
                          outdir=path_to_save,check_point=False,
                         resume=False,dlogz=0.1,clean=True,label=label_name)
```



How to detect?

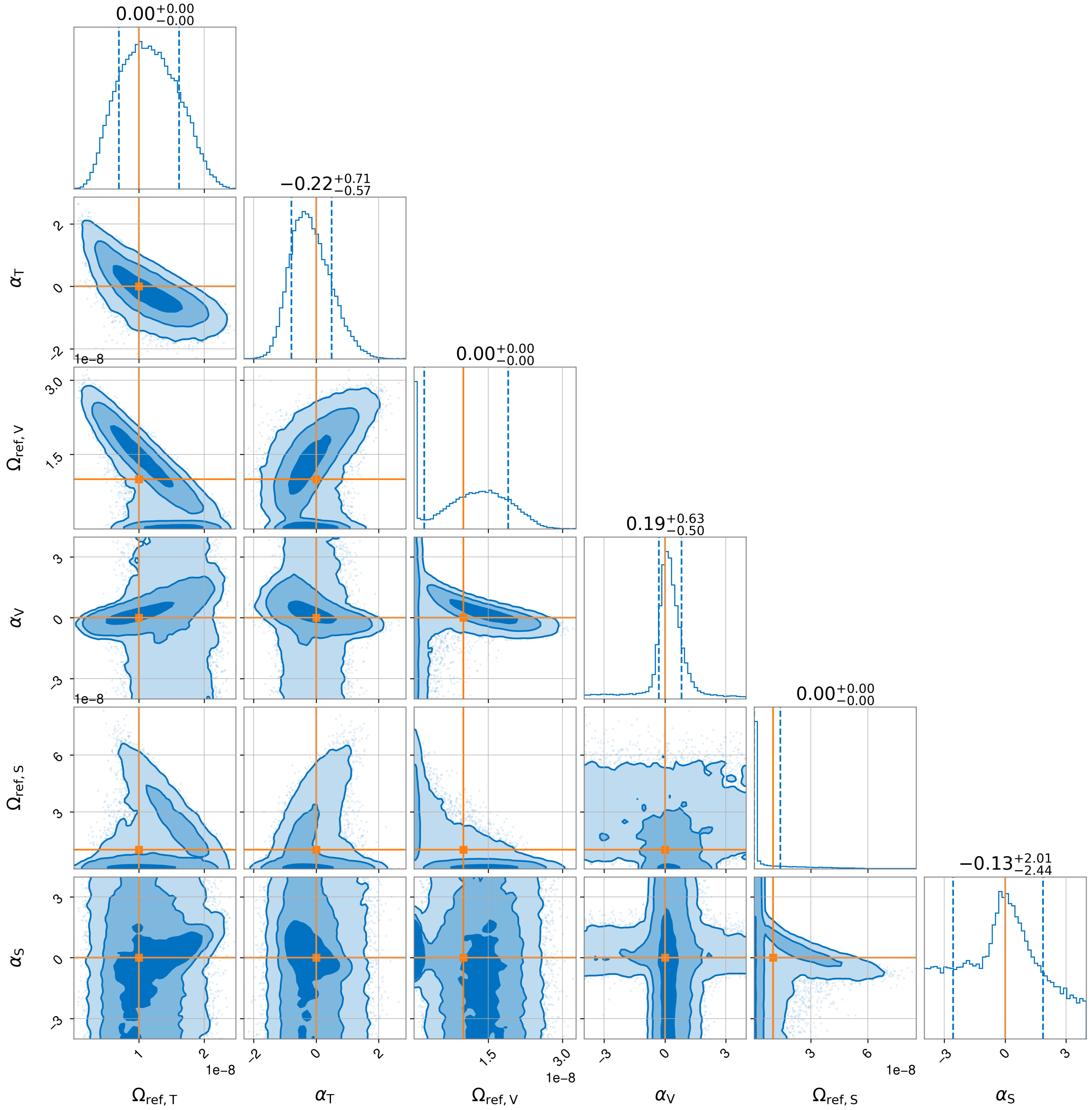
$$p(\hat{\Omega}_0|\Theta,\Lambda) \propto \exp\left[-\frac{1}{2}\sum_f \frac{(\hat{\Omega}_0(f) - \Lambda\Omega_{\rm GW}(f;\Theta))^2}{\sigma_0^2(f)}\right]$$

Non - GR polarization

```
kwargs = {"baselines":[HL], "model_name":'TVSPowerLawModel', "polarizations":['tensor', 'vector', 'scalar'], "fref": 25}
model= pe.TVSPowerLawModel(**kwargs)
priors = {'omega_ref_tensor': bilby.core.prior.LogUniform(10**(-6), 10**(-2), '$\\Omega_{\\rm ref, T}$'),
          'alpha_tensor': bilby.core.prior.Uniform(-4, 4, '$\\alpha_{\\rm T}$'),
          'omega_ref_vector': bilby.core.prior.LogUniform(10**(-6), 10**(-2), '$\\Omega_{\\rm ref, V}$'),
          'alpha_vector': bilby.core.prior.Uniform(-4, 4, '$\\alpha_{\\rm V}$'),
          'omega_ref_scalar': bilby.core.prior.LogUniform(10**(-6), 10**(-2), '$\\Omega_{\\rm ref, S}$'),
          'alpha_scalar': bilby.core.prior.Uniform(-4, 4, '$\\alpha_{\\rm S}$')}

result=bilby.run_sampler(likelihood=model,priors=priors,sampler='dynesty', npoints=10000, walks=10, npool=5,
                         outdir=outdir,label= 'hlv_result_non_GR_pols_chh', resume=False,clean=True,dlogz=0.1)
```

```
08:27 bilby INFO    : Summary of results:
nsamples: 27617
ln_noise_evidence:  0.000
ln_evidence: 11834.865 +/-  0.031
ln_bayes_factor: 11834.865 +/-  0.031
```



Take a break now, and be ready for Hand on section of PyGWB