



Introduction to Calibration of LIGO detectors

Dripta Bhattacharjee

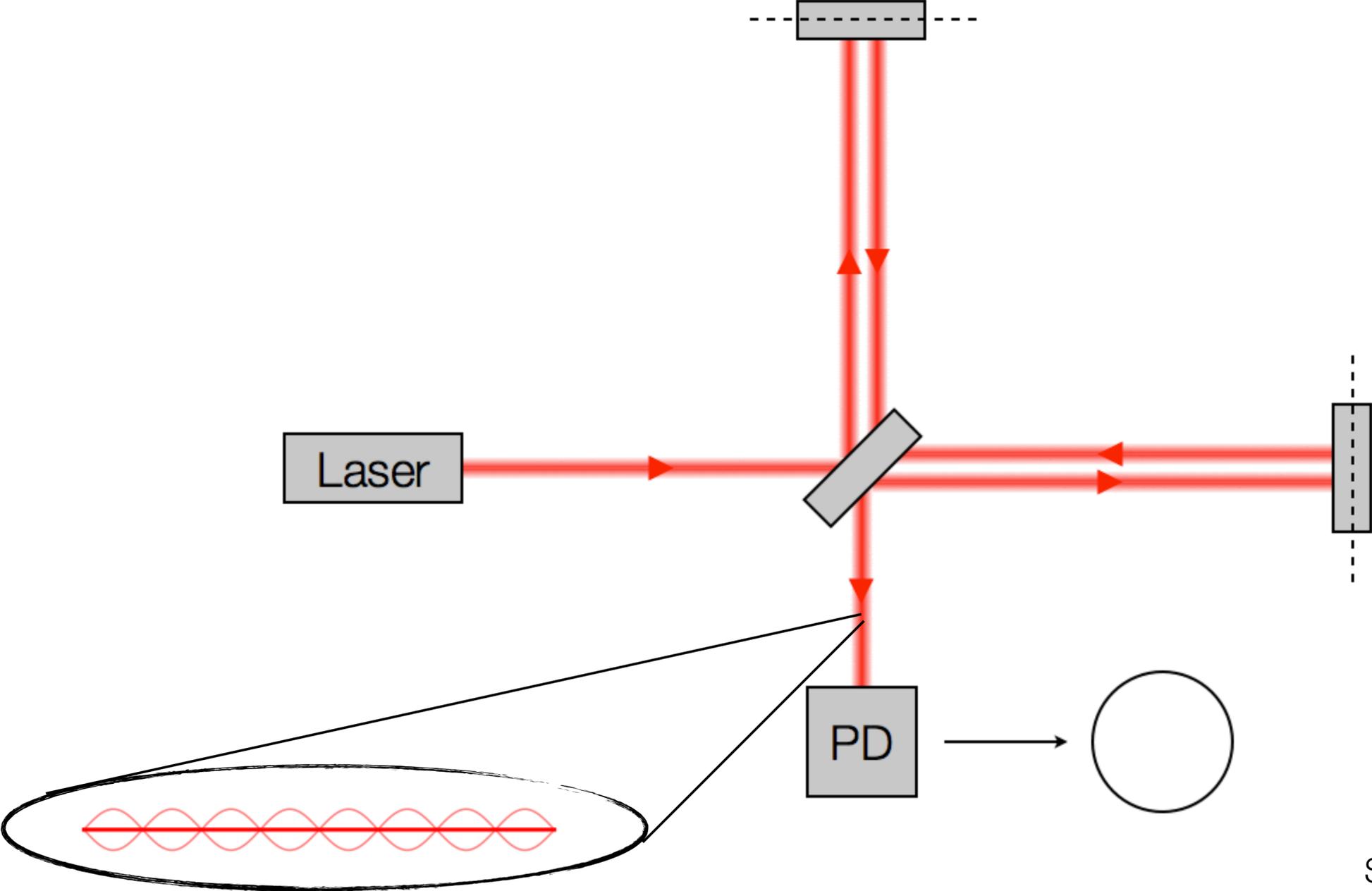
What is calibration?

Why is it important?

Absolute calibration

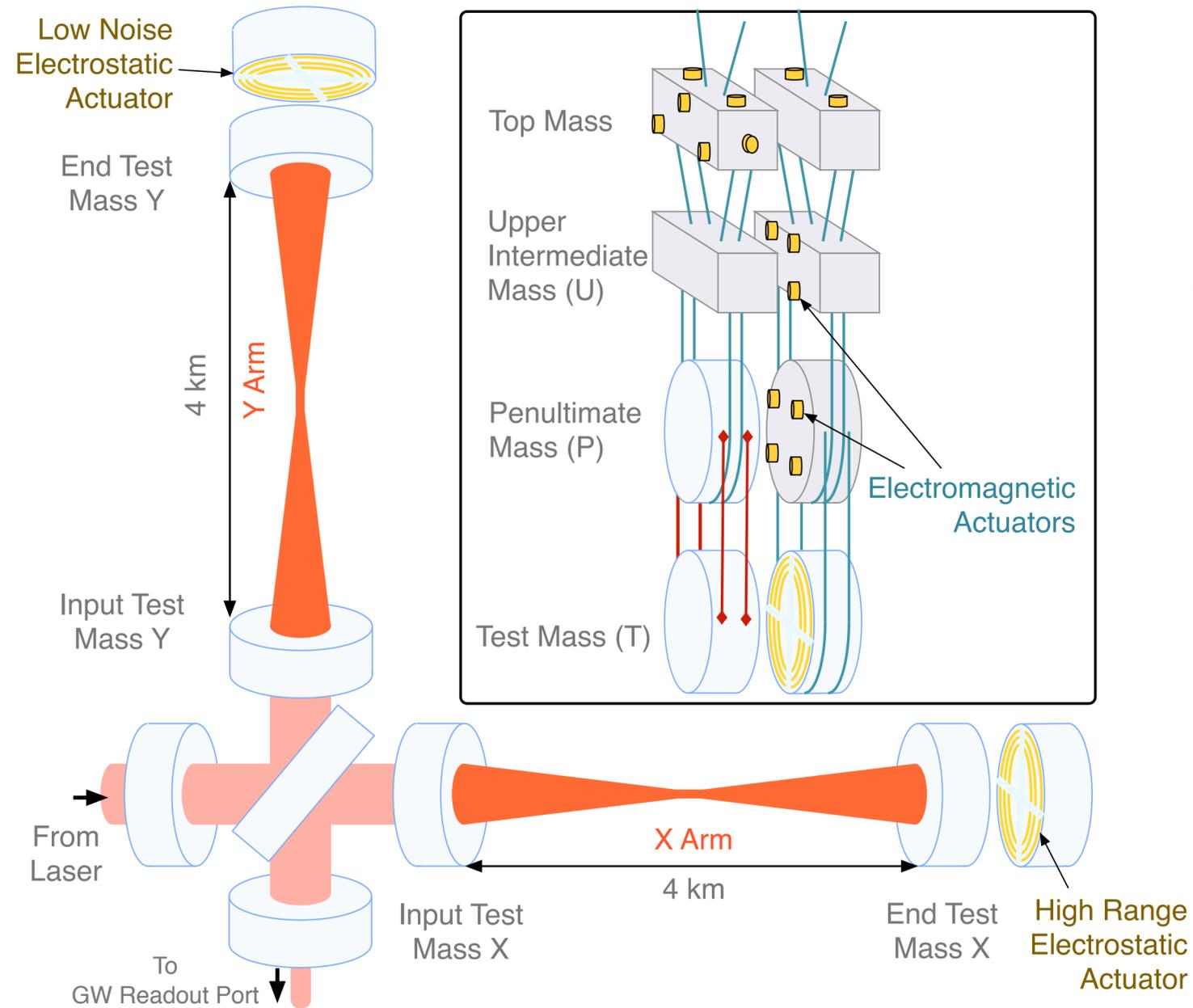
Interferometer calibration

Calibration: Converting PD output to strain



Slide by Madeline Wade

Basic LIGO interferometric schematic



$$\Delta L_{\text{free}} = \Delta L_x(t) - \Delta L_y(t)$$

$$h(t) = \frac{\Delta L_{\text{free}}(t)}{L}$$

Without calibration there is no $h(t)$

Phys. Rev. D 96, 102001

What is calibration?

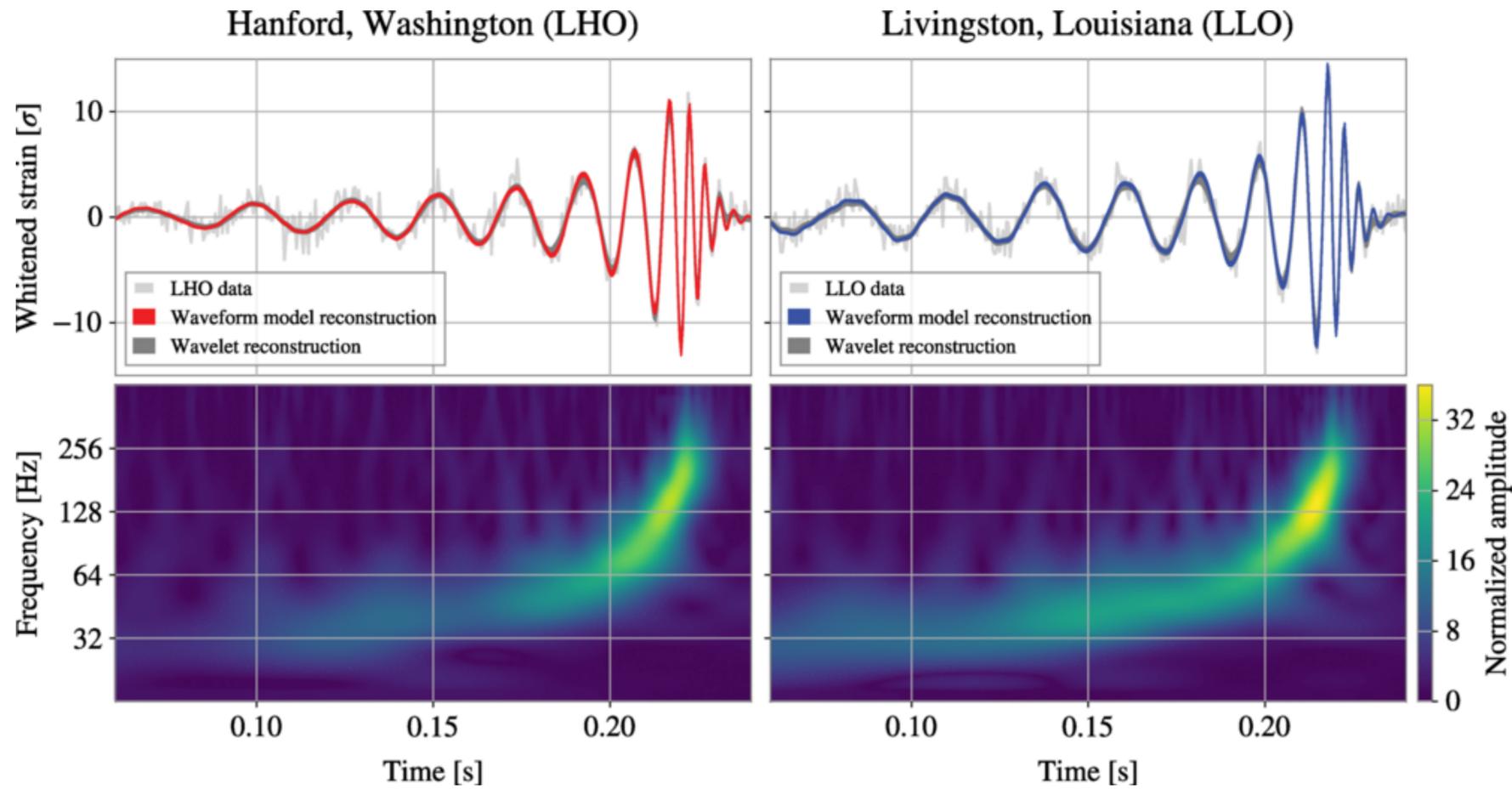
Why is it important?

Absolute calibration

Interferometer calibration

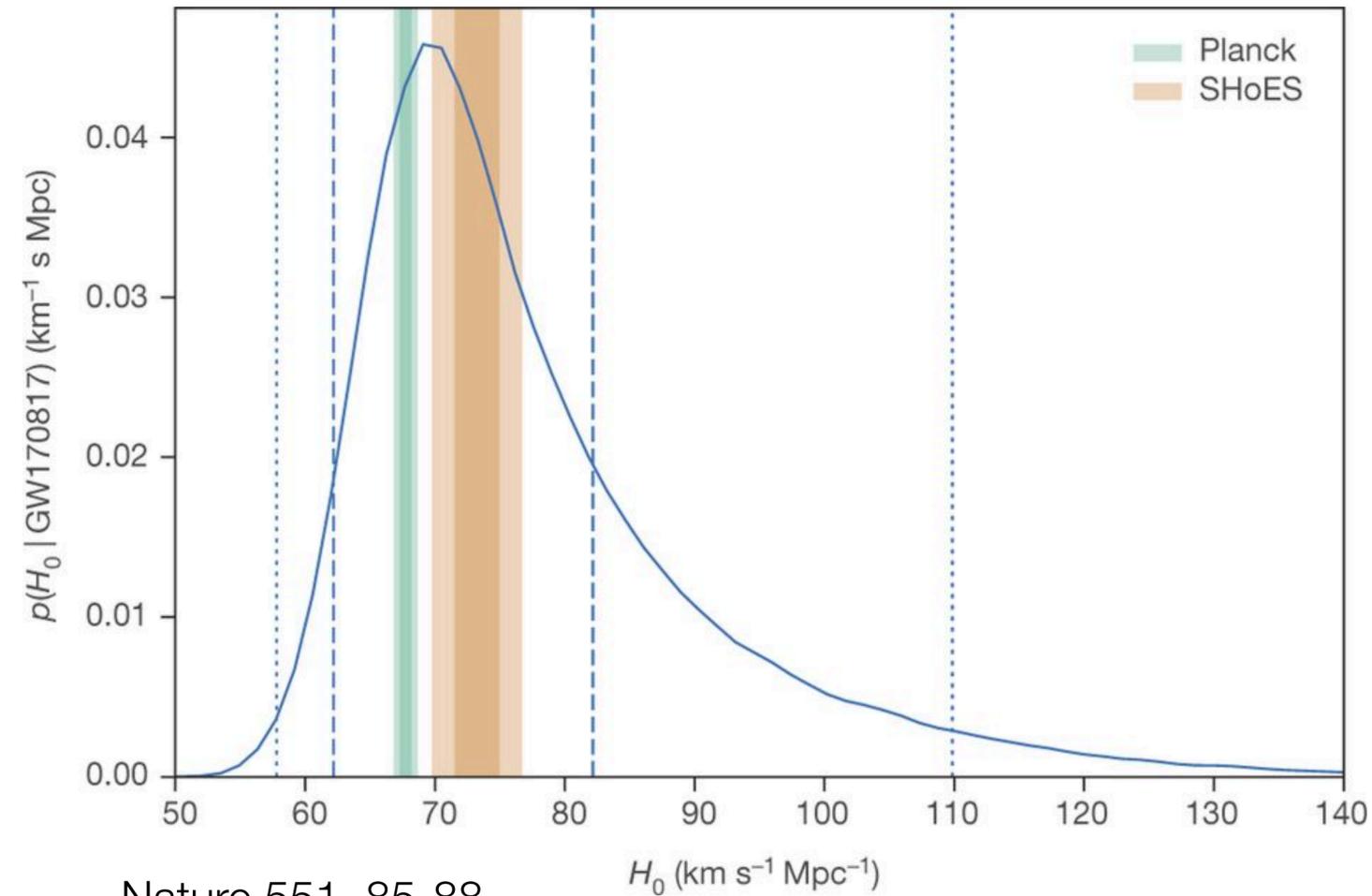
Science enabled by accurate calibration

Testing Hawking's Area Law: GW250114 with network SNR ~ 80



Phys. Rev. Lett. 135, 111403

GW170817: Independent measurement of Hubble parameter

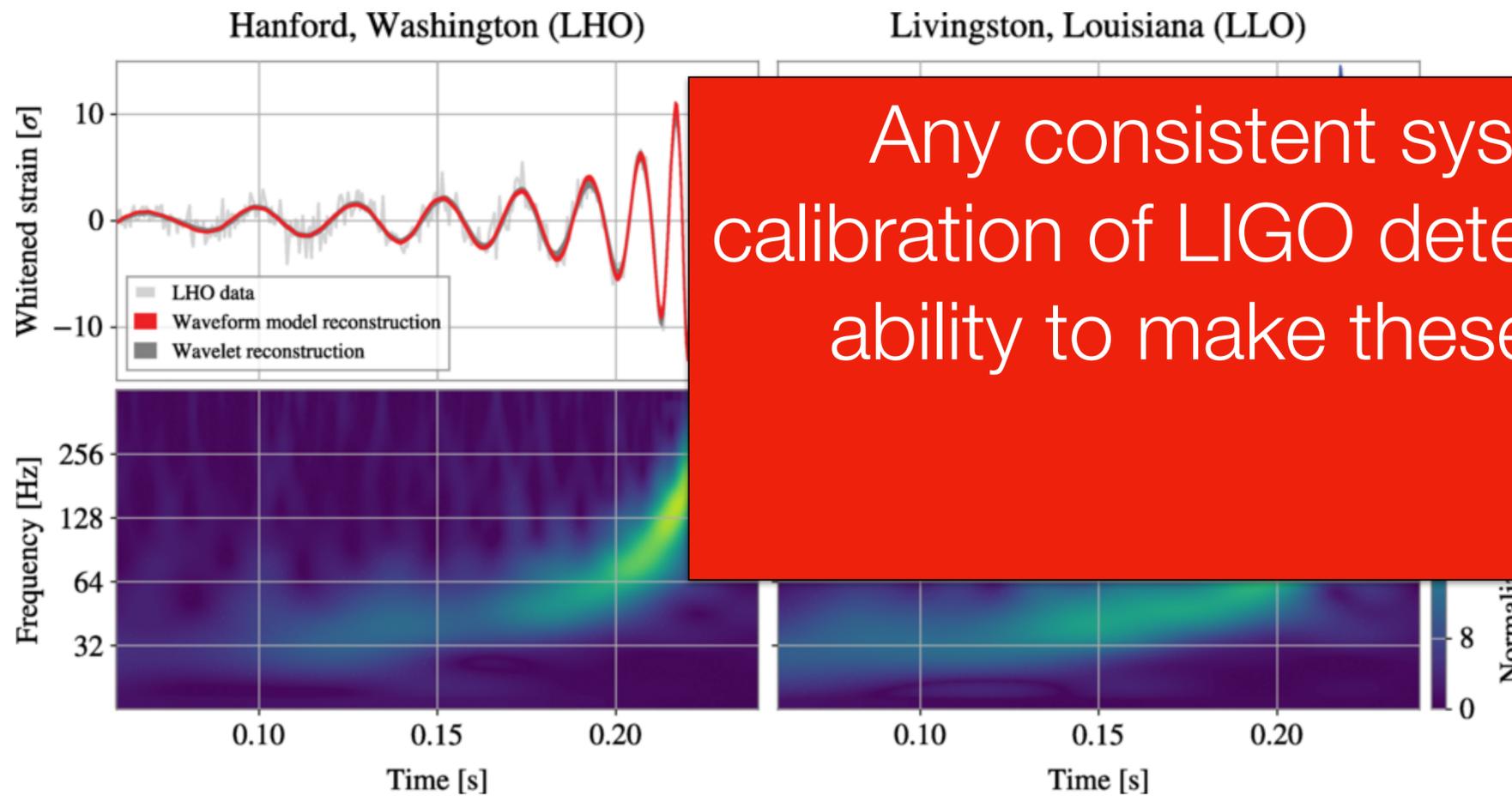


Nature 551, 85-88

Science enabled by accurate calibration

Testing Hawking's Area Law: GW250114 with network SNR ~ 80

Independent measurement of Hubble parameter



Any consistent systematic error in calibration of LIGO detectors will inhibit our ability to make these measurements



Phys. Rev. Lett. 135, 111403

Nature 551, 85-88

H_0 ($\text{km s}^{-1} \text{Mpc}^{-1}$)

What is calibration?

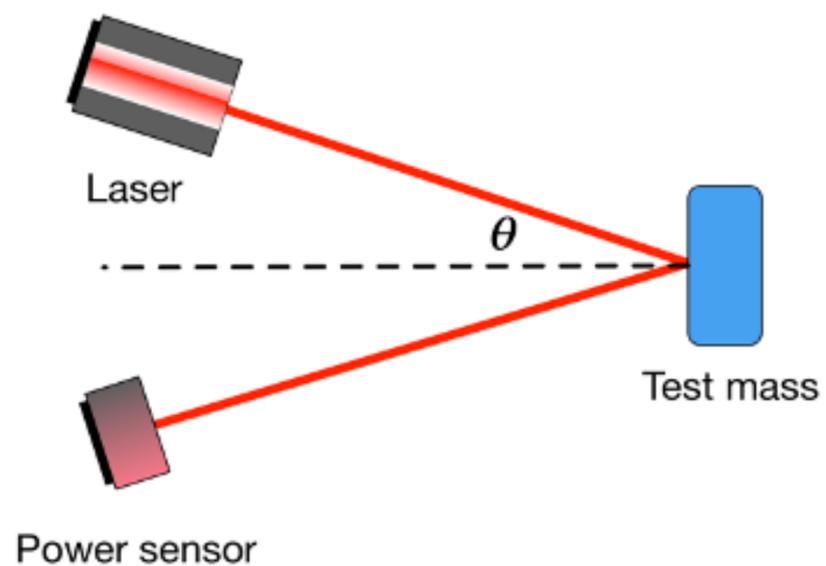
Why is it important?

Absolute calibration

Interferometer calibration

How do we calibrate length variations? Photon calibrators

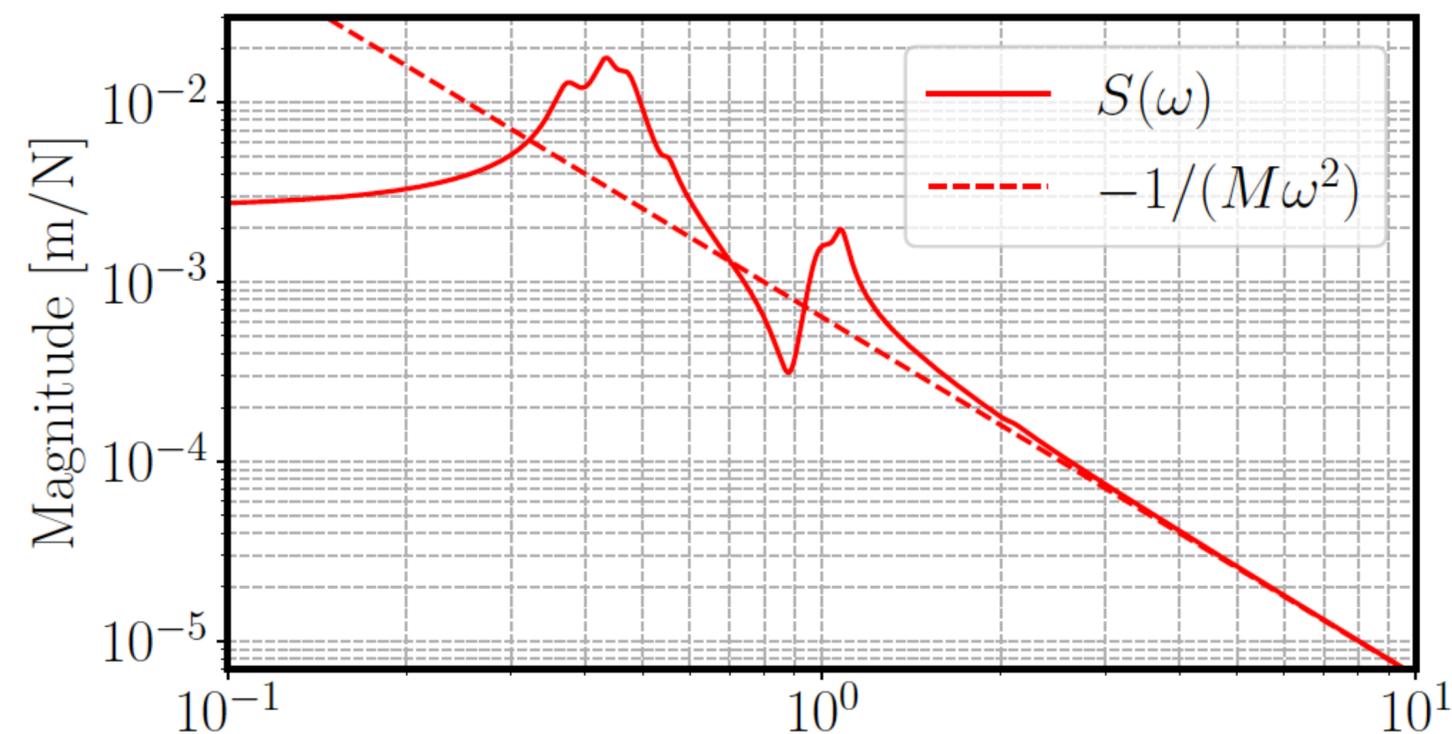
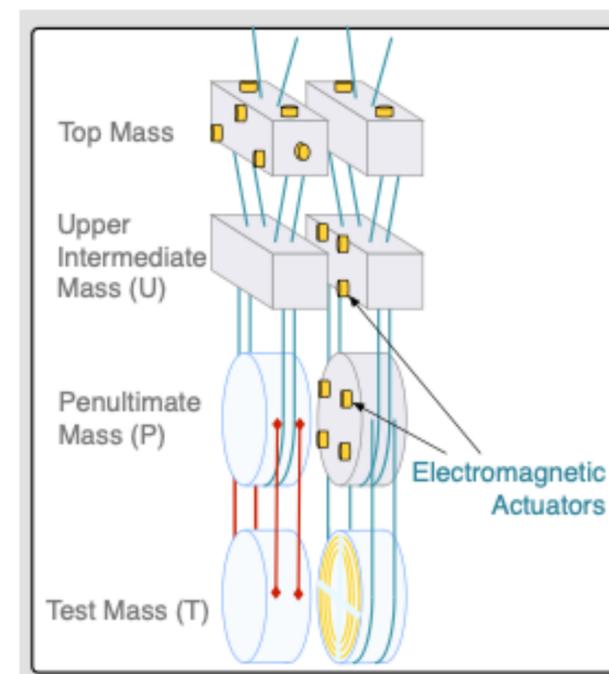
Known force exerted by a reflecting beam



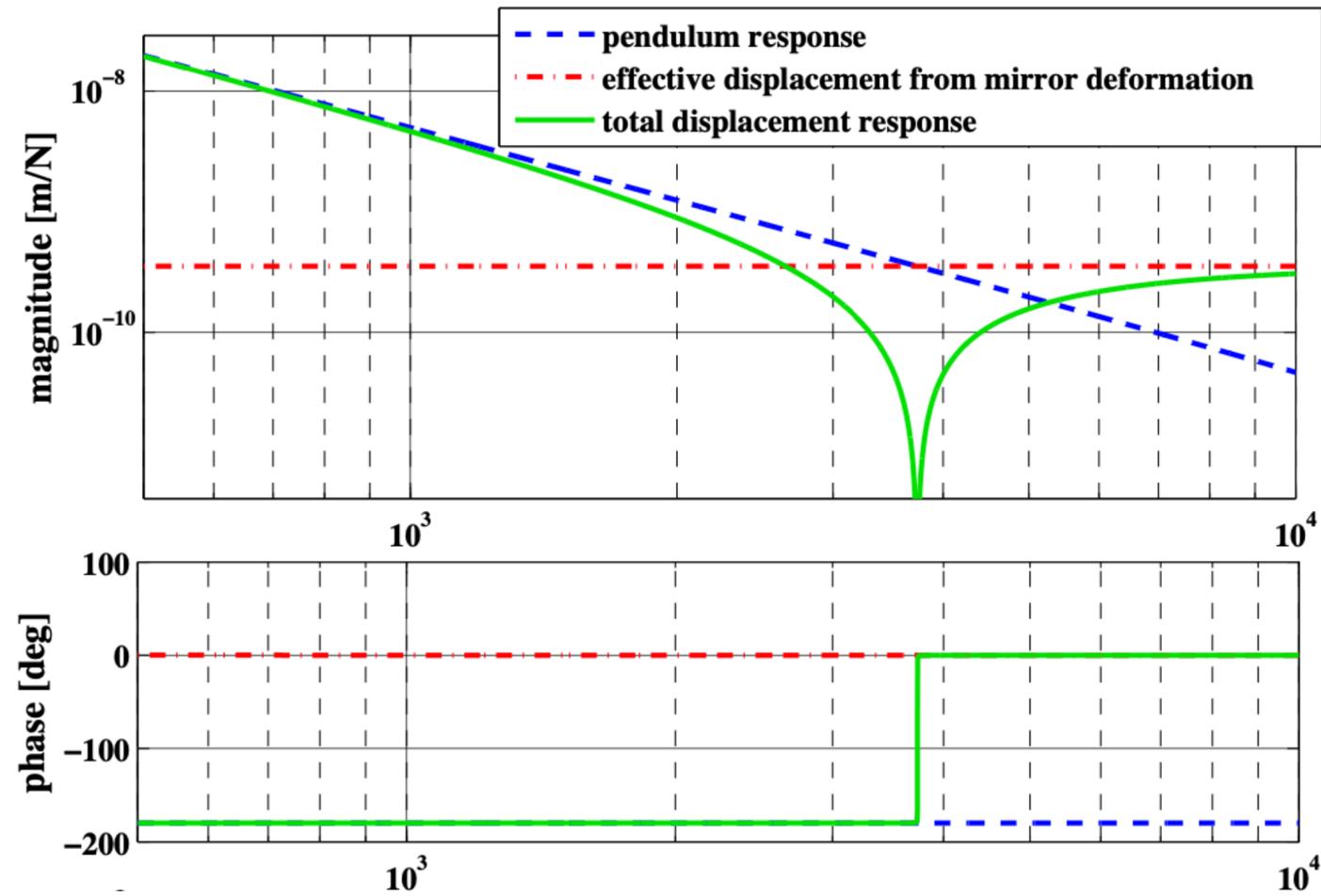
$$F(\omega) = \frac{2 \cos \theta}{c} P(\omega)$$

For a free mass:

$$x(\omega) = - \frac{2 \cos \theta}{M \omega^2 c} P(\omega)$$

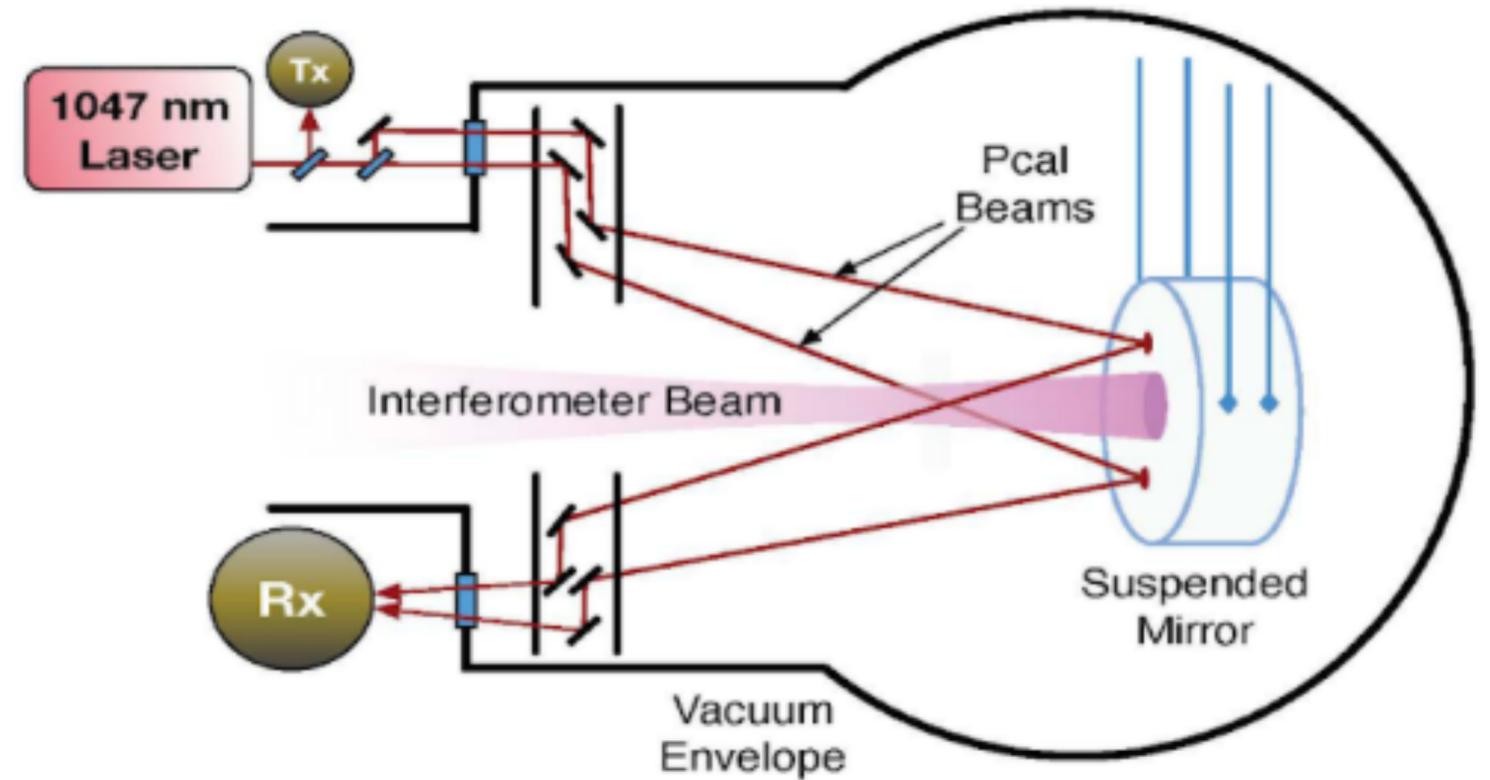


But, local elastic deformations ...



S. Hild, et al., Class. Quantum Grav. 24 (2007) 5681

Use two beams:



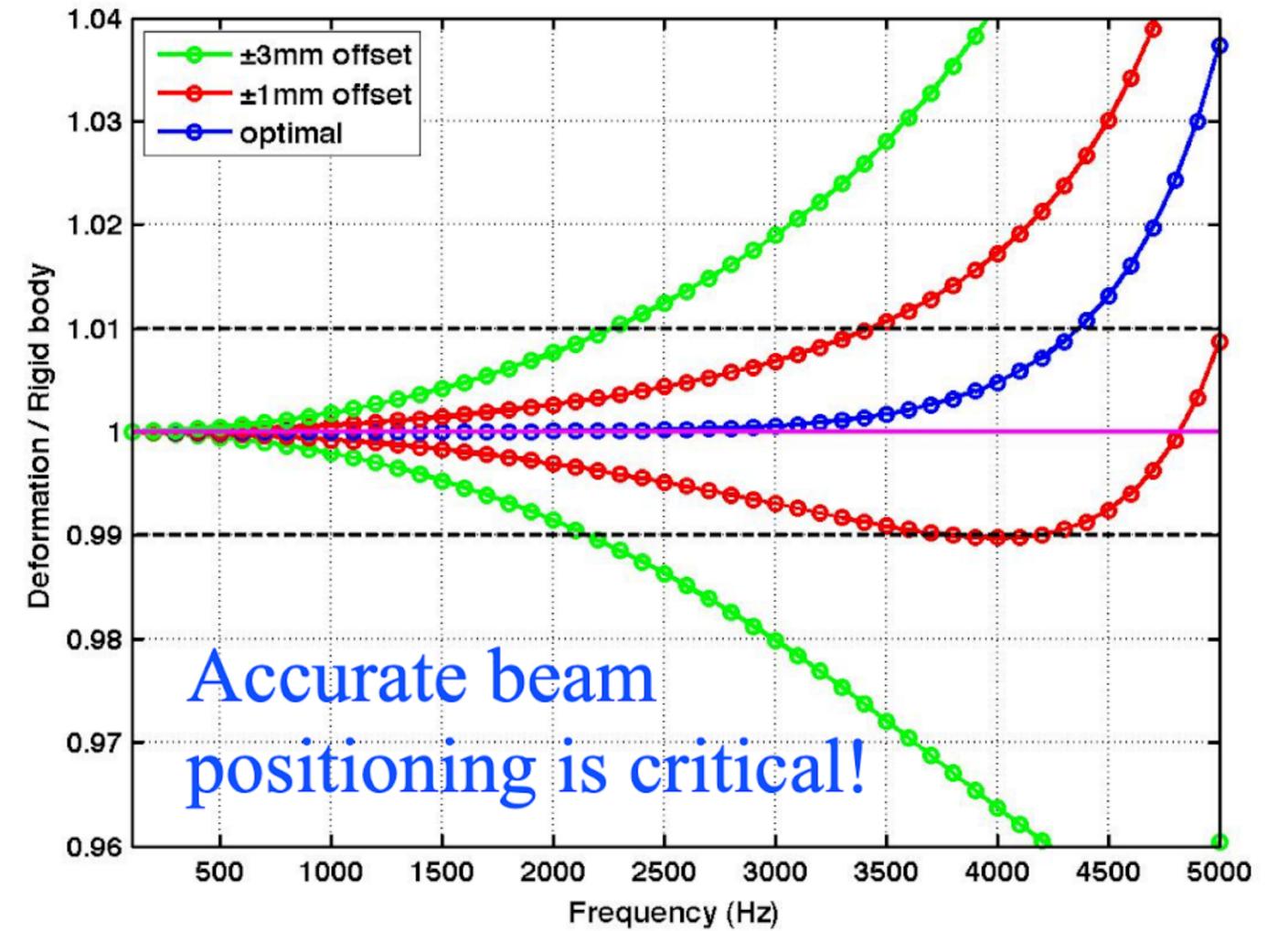
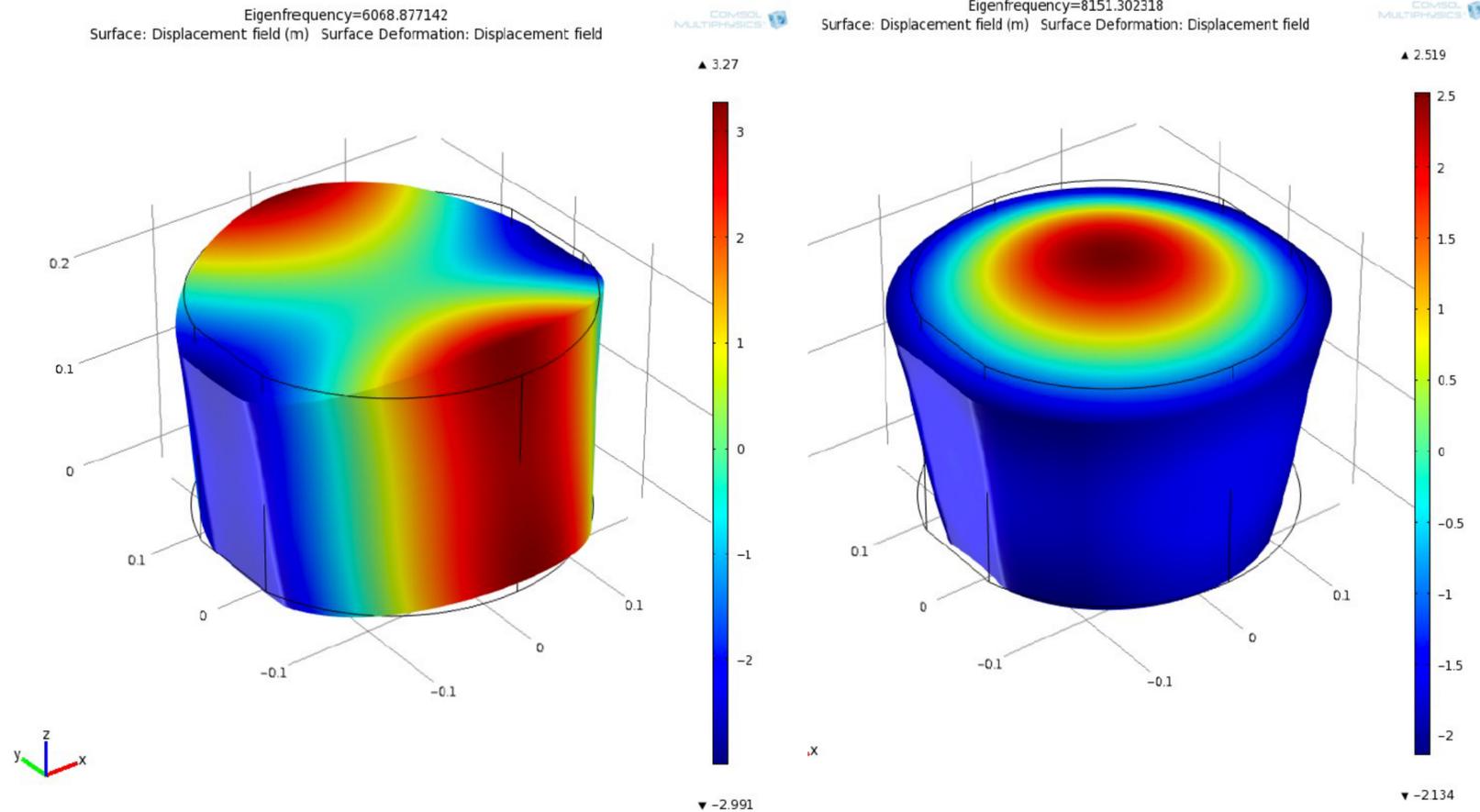
Use two beams, diametrically opposed away from centre of the mirror

E. Goetz, et al. Class. Quantum Grav. 26 (2009) 245011

Beam locations

“Butterfly” mode ~6 kHz

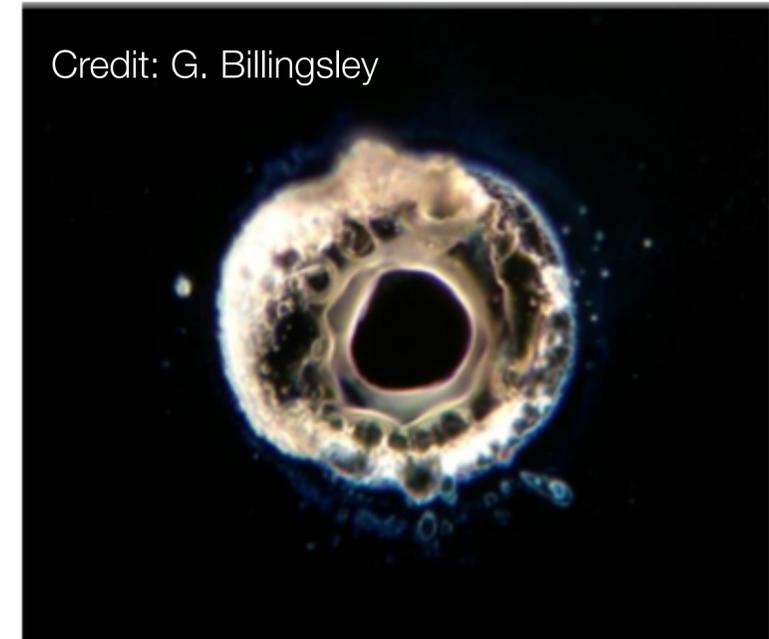
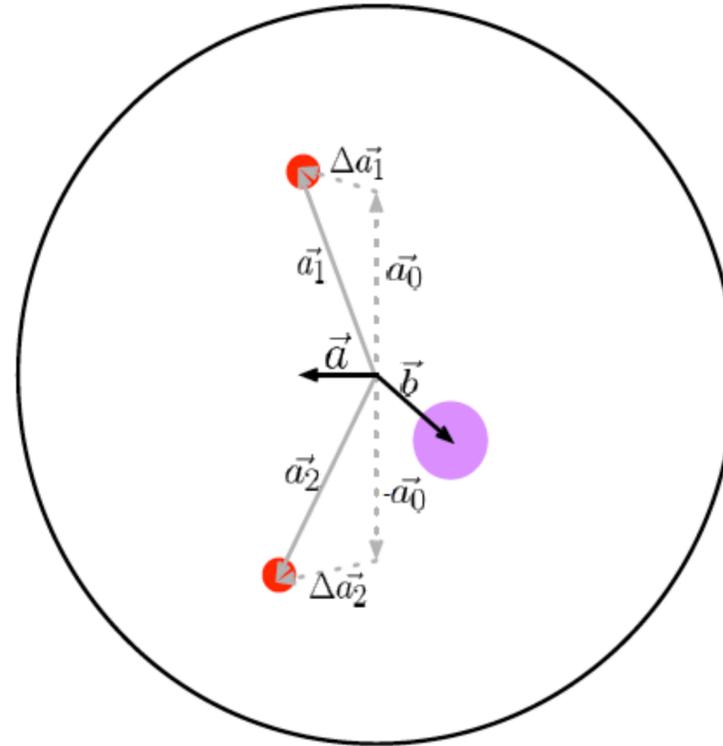
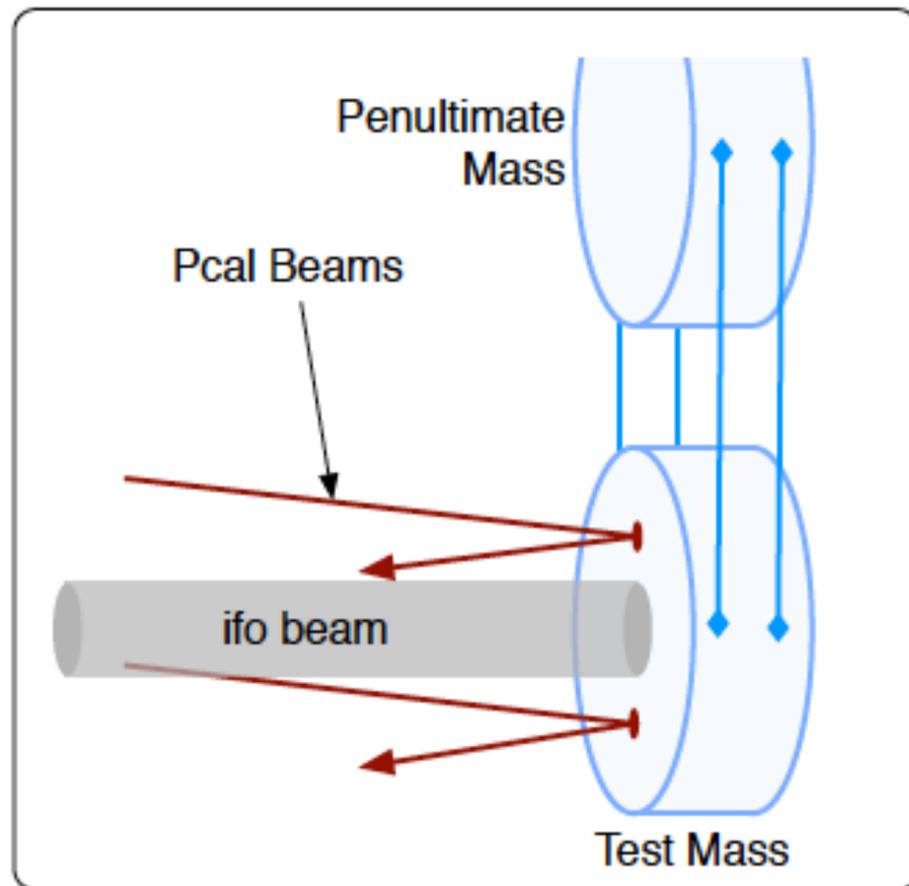
“Drumhead” mode ~8 kHz



- M. Afrin Badhan et al. LIGO T090401 (2009)
- P. Daveloza, et al. LIGO-G1200900 (2012)
- N. De Lillo and S. Kandhasamy LIGO-T1700213 (2017)
- S. Karki LIGO-P1900217 (2019)

Locate beams close to nodal circle for “Drumhead” mode

And unintended rotation ...



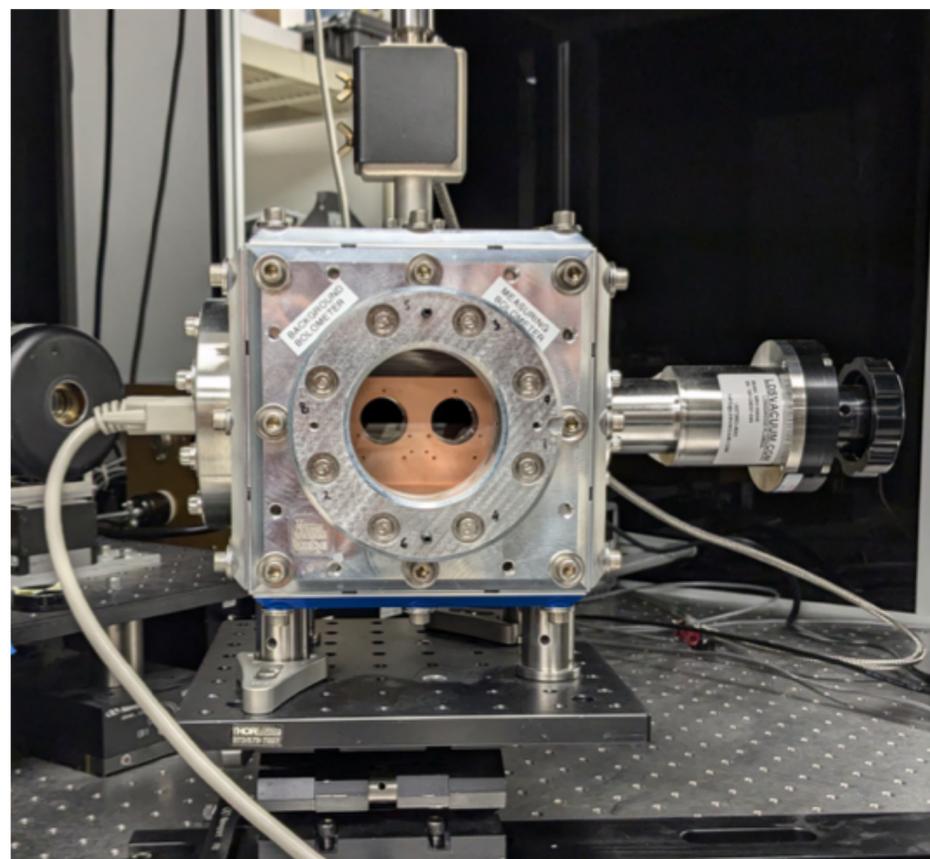
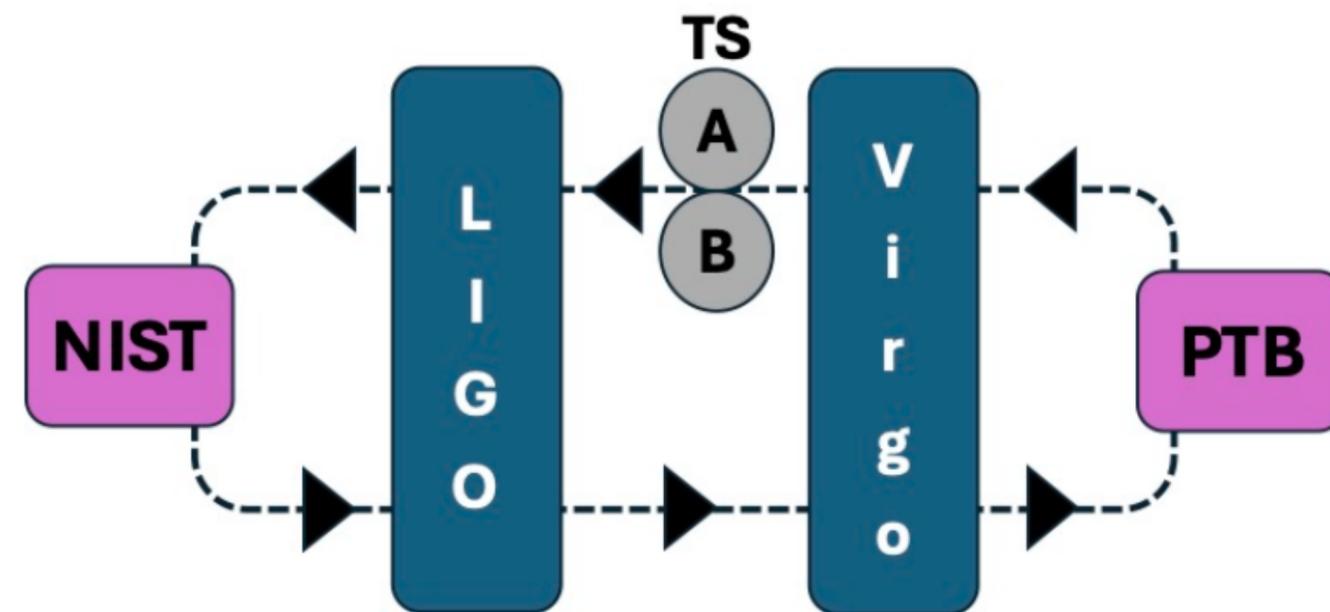
Credit: G. Billingsley

$$x(\omega) \simeq -\frac{2 \cos \theta}{Mc \omega^2} P(\omega) \left[1 + \frac{M}{I} (\vec{a} \cdot \vec{b}) \right]$$

Accurate and precise power estimate
(on ETM) provide accurate and precise
displacement calibration

Uncertainty in Pcal-induced
displacement

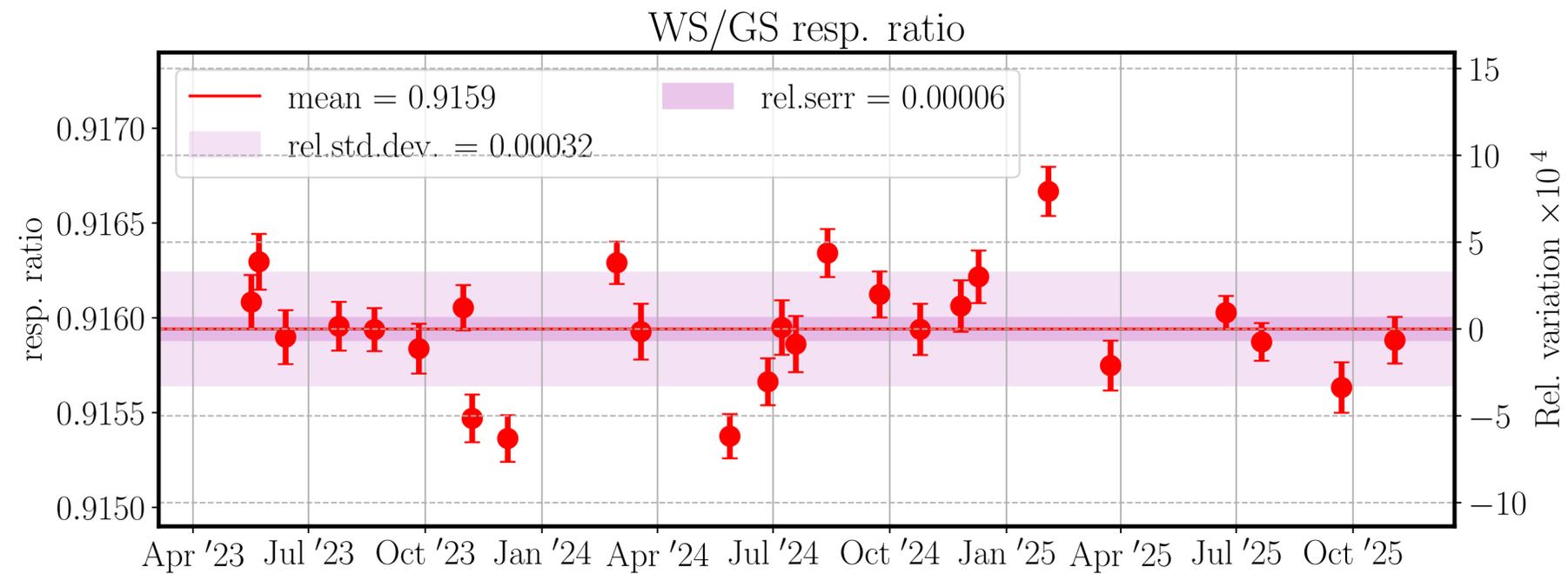
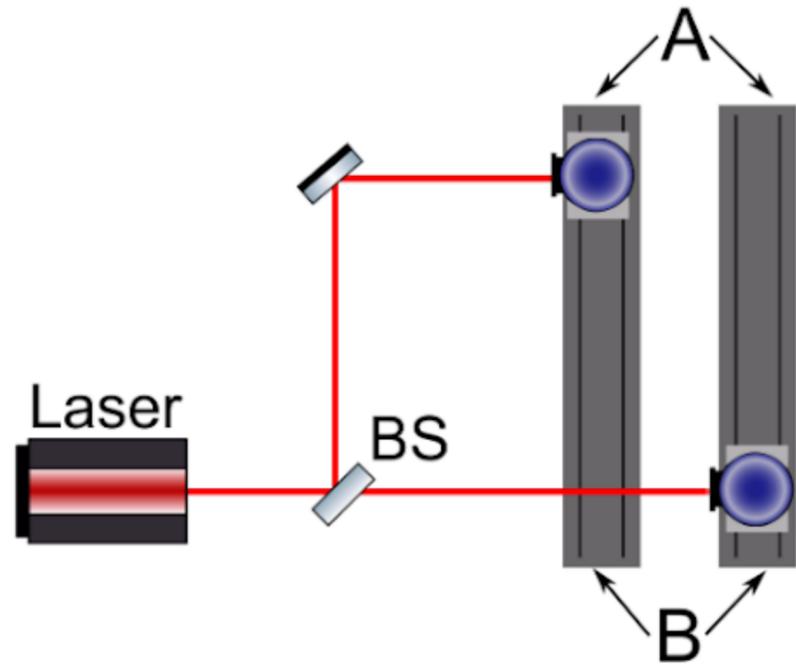
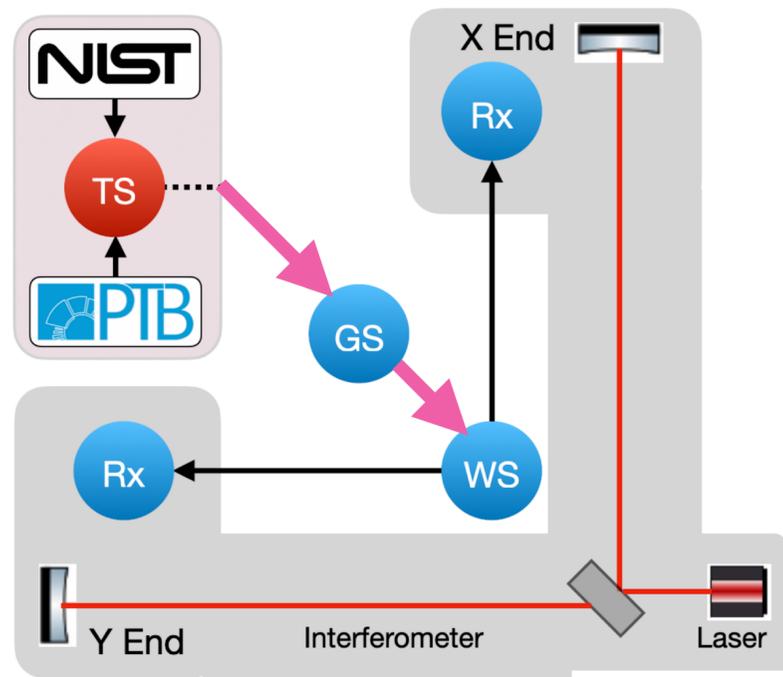
Power sensor calibration



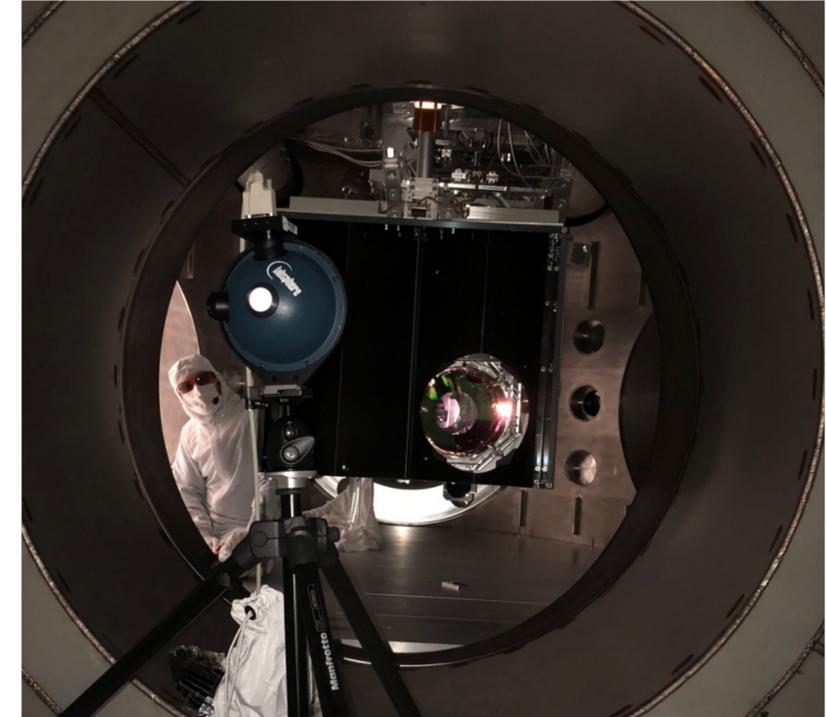
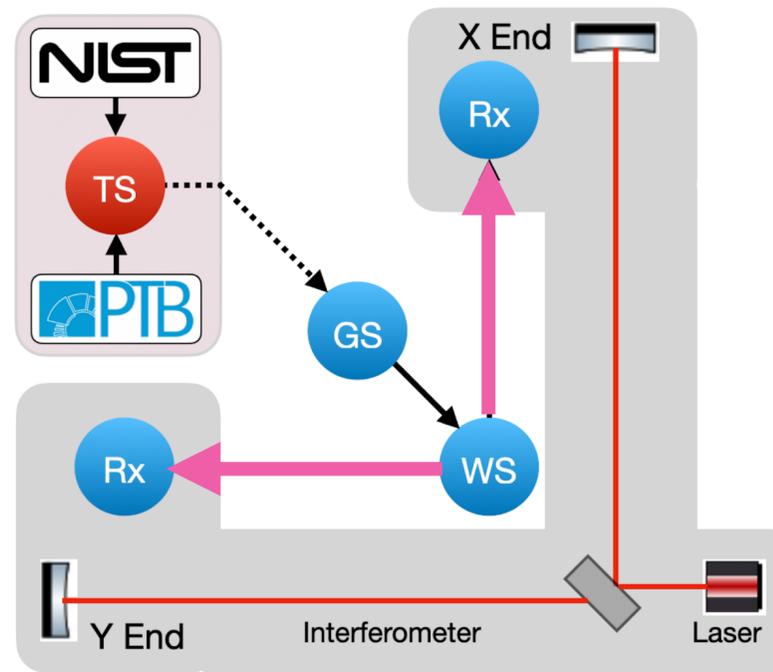
A.Vaskuri, et al., Opt. Express 29 (2021) 22533-52



Power sensor calibration propagation at Pcal lab

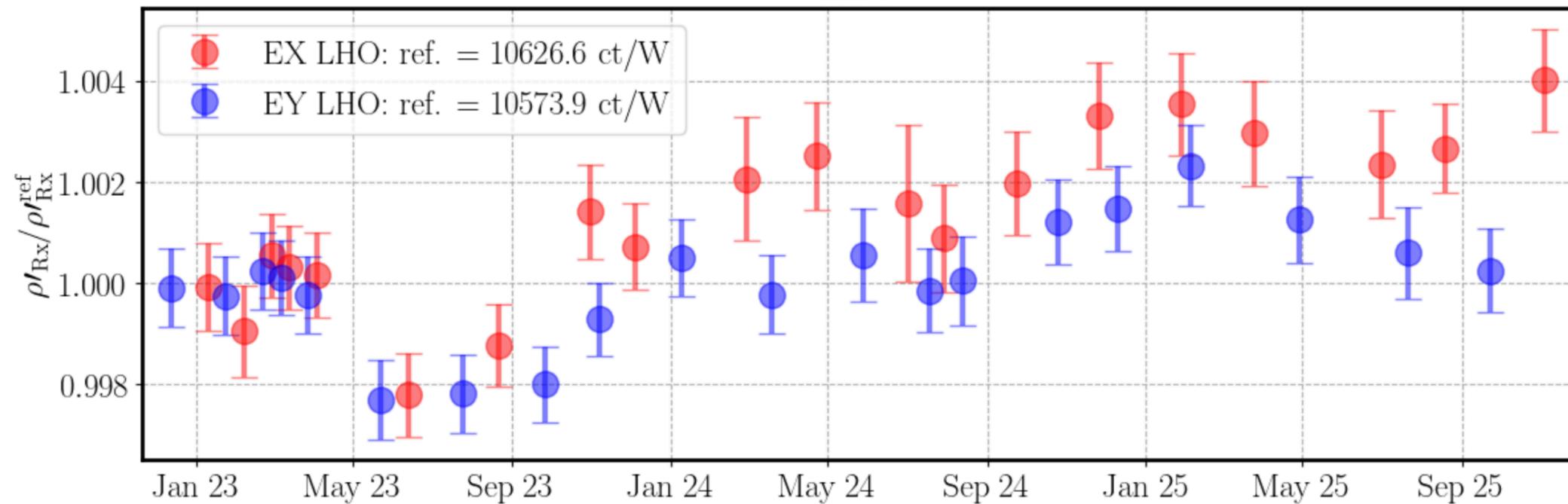


Power sensor calibration propagation to end station sensors



R. Savage

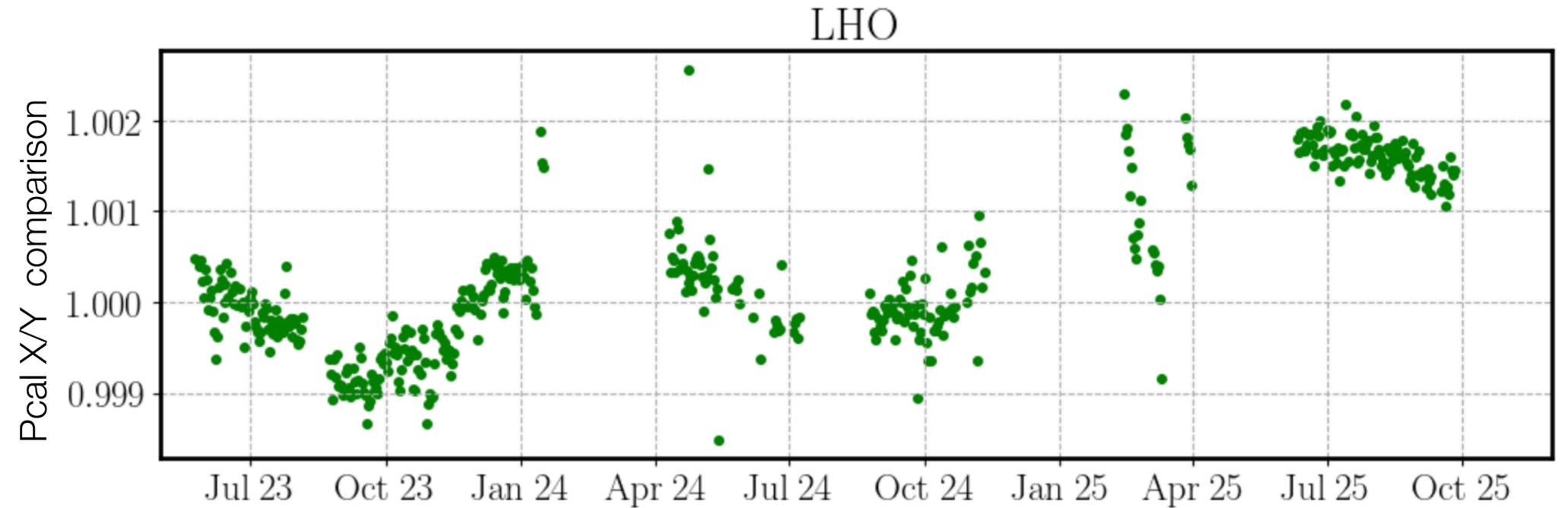
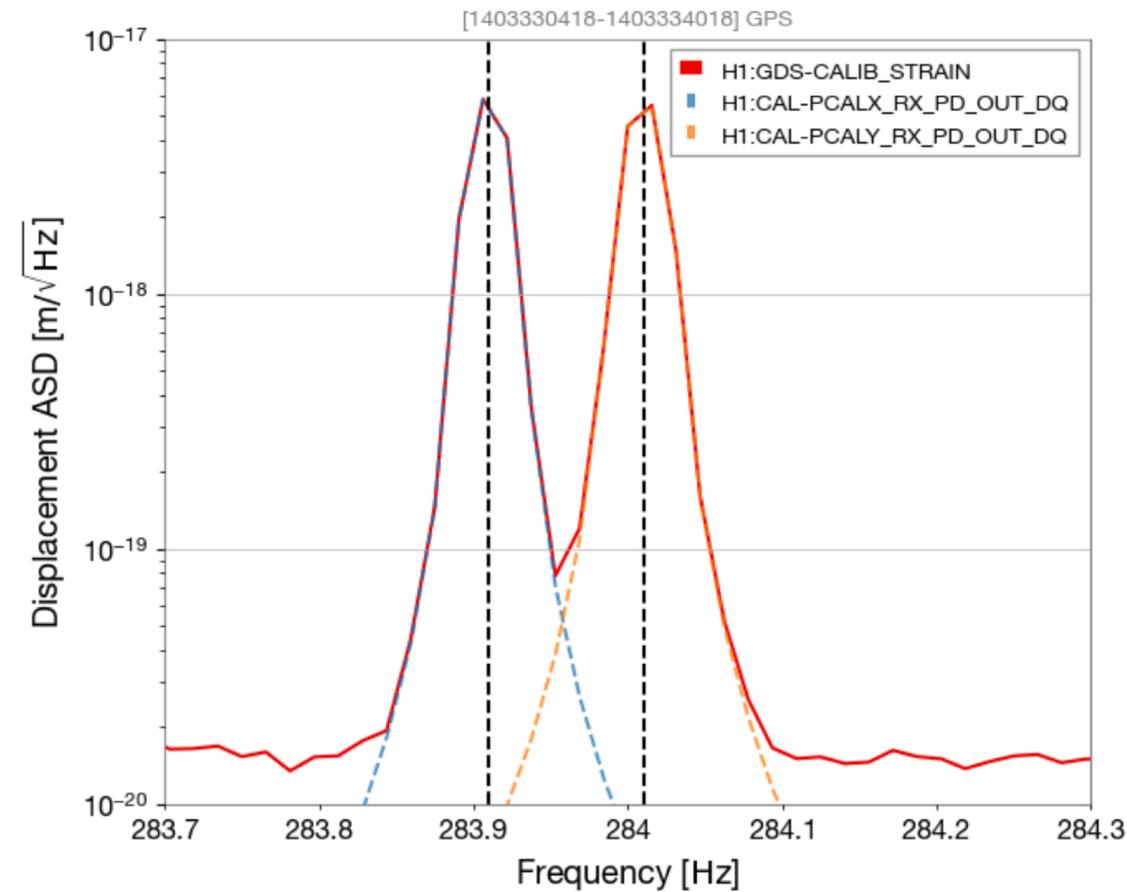
LHO



Unknown and unaccounted systematic error at both end stations - under investigation

Both seem to follow each other - indication of temperature dependence

Temporal variations in Pcal-induced displacement fiducials



This method tells us relative Pcal calibration variations - NOT the common part

Slight detour: Virgo used Ncals to calibrate their ifo during O4. Adding a Ncal to the LIGO system, even if uncalibrated, will help in investigating unknown systematics.

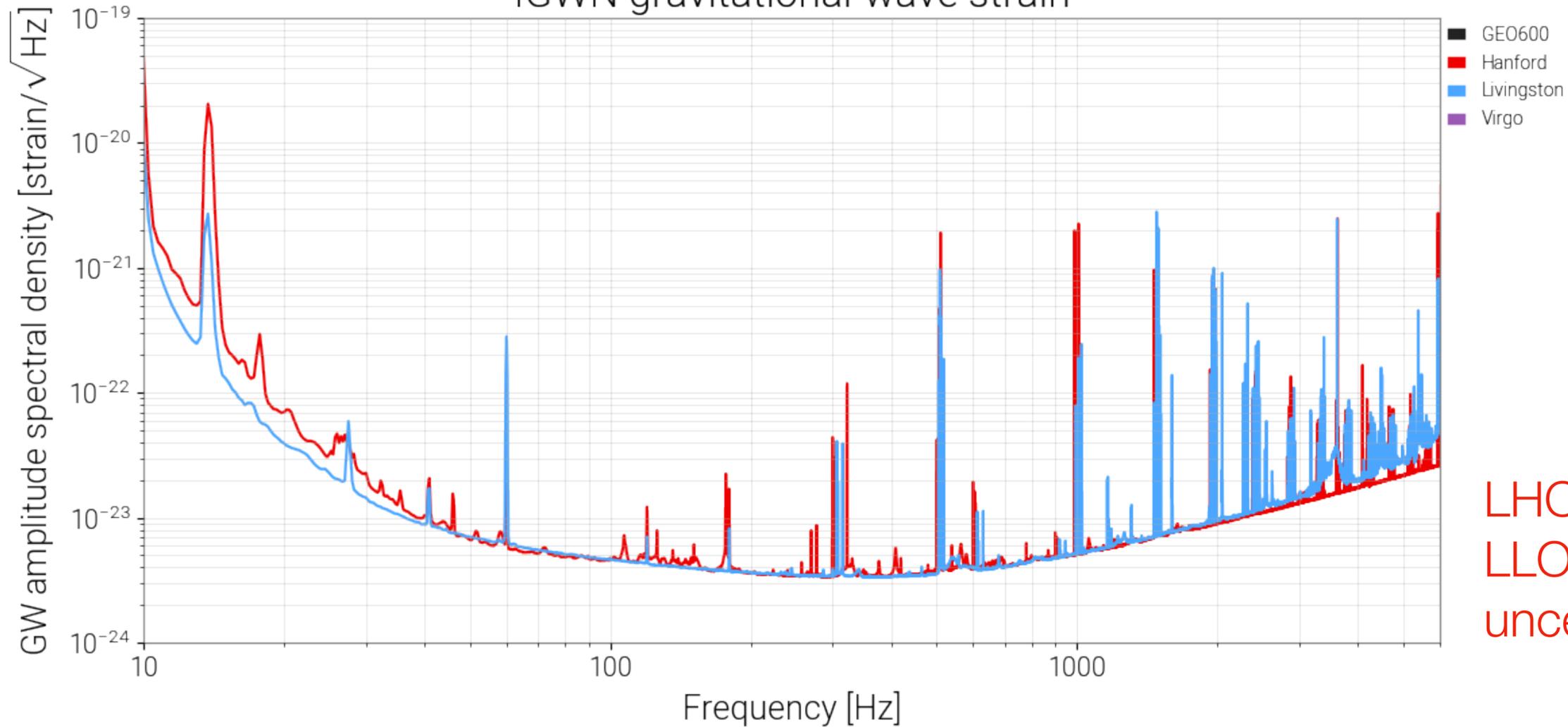
D. Bhattacharjee, et al. Class. Quantum Grav. 38 (2021) 015009

Calibrated displacement fiducials

$$x(\omega) \simeq -\frac{2 \cos \theta}{Mc \omega^2} P(\omega) \left[1 + \frac{M}{I} (\vec{a} \cdot \vec{b}) \right]$$

[1442707218-1442793618, state: Locked]

IGWN gravitational-wave strain



LHO: 0.29%

LLO: 0.15%

LHO has higher uncertainty than LLO due to larger rotational uncertainty

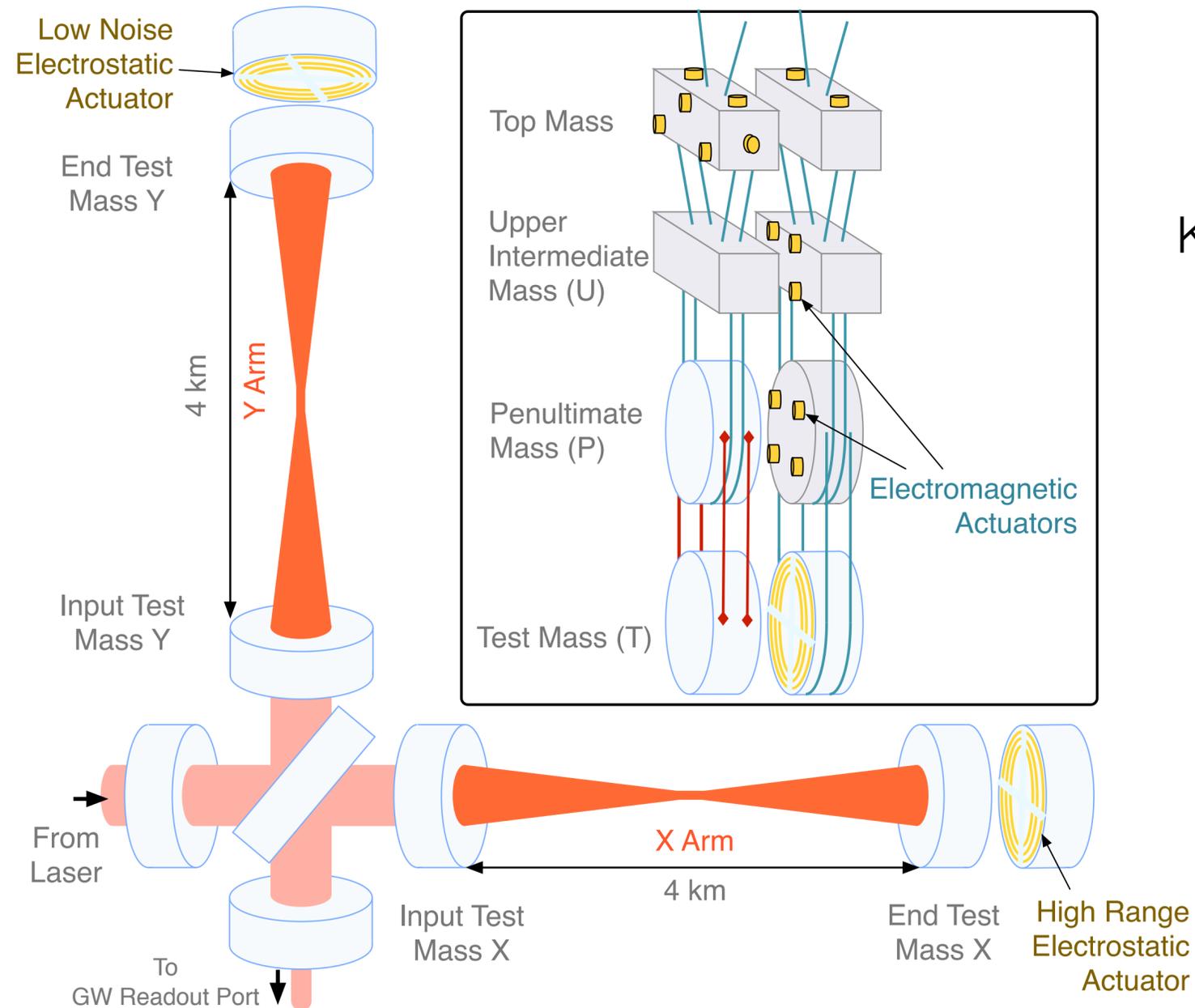
What is calibration?

Why is it important?

Absolute calibration

Interferometer calibration

Back to basic LIGO interferometric schematic



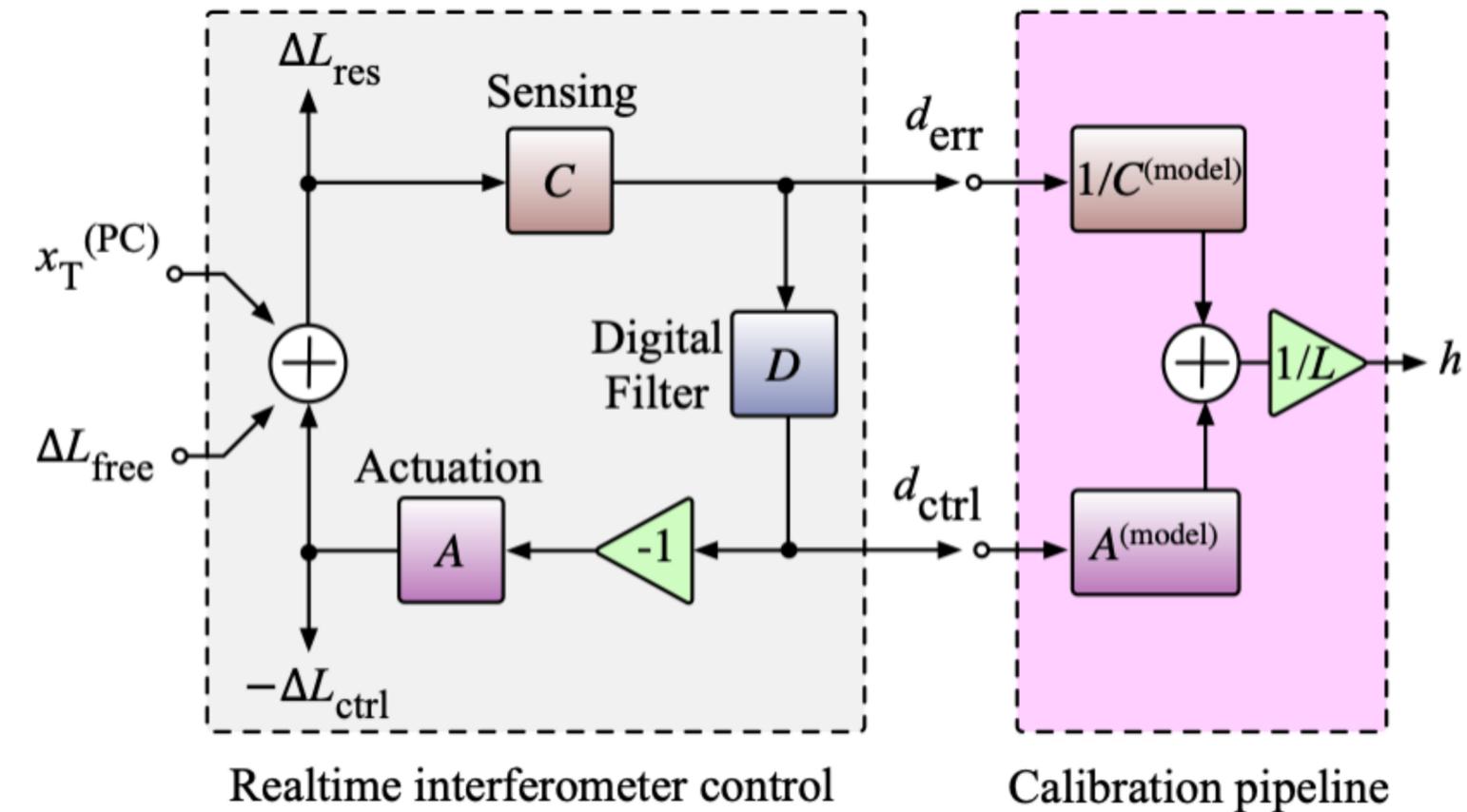
Keeping the detector “locked” involves:

Lots of active seismic isolation
(including quad. pendulum system)

AND control loop applied to readout
signal

Phys. Rev. D 96, 102001

DARM loop: reconstructing $h(t)$



To reconstruct the timeseries for strain

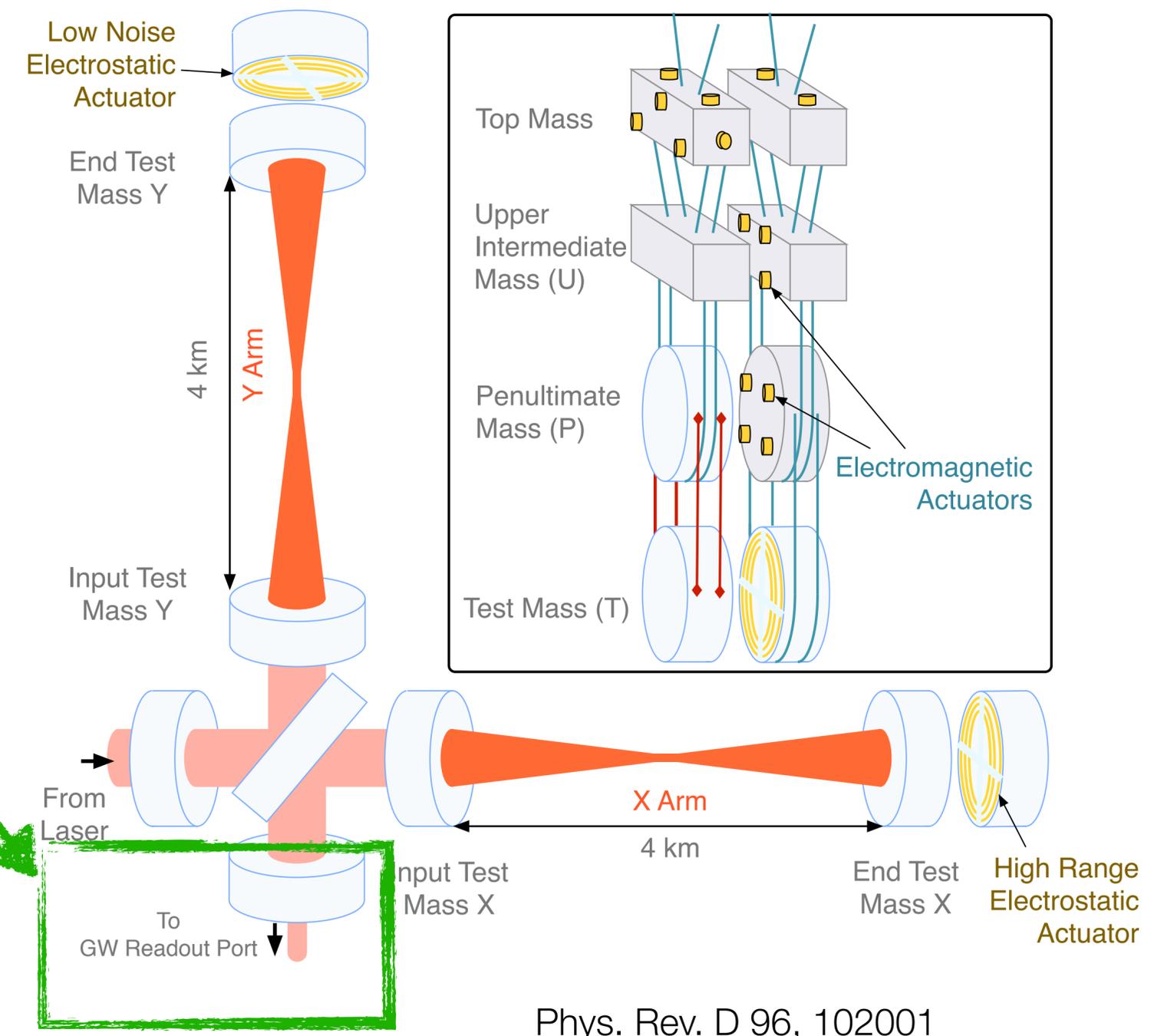
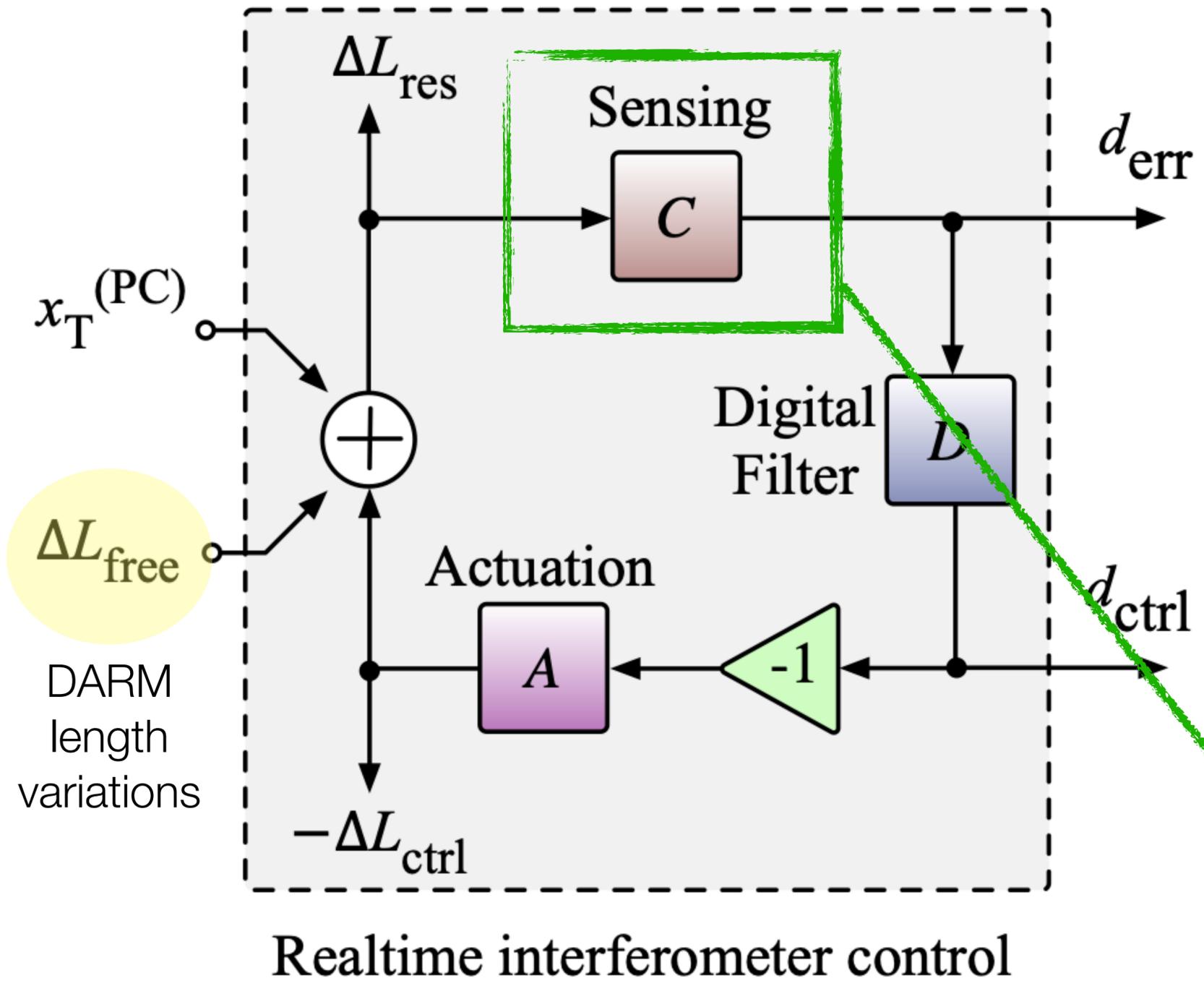
$$h(t) L = \frac{1}{C} * d_{\text{err}} + A * d_{\text{ctrl}}$$

$$h(t) = R * d_{\text{err}}(t)$$

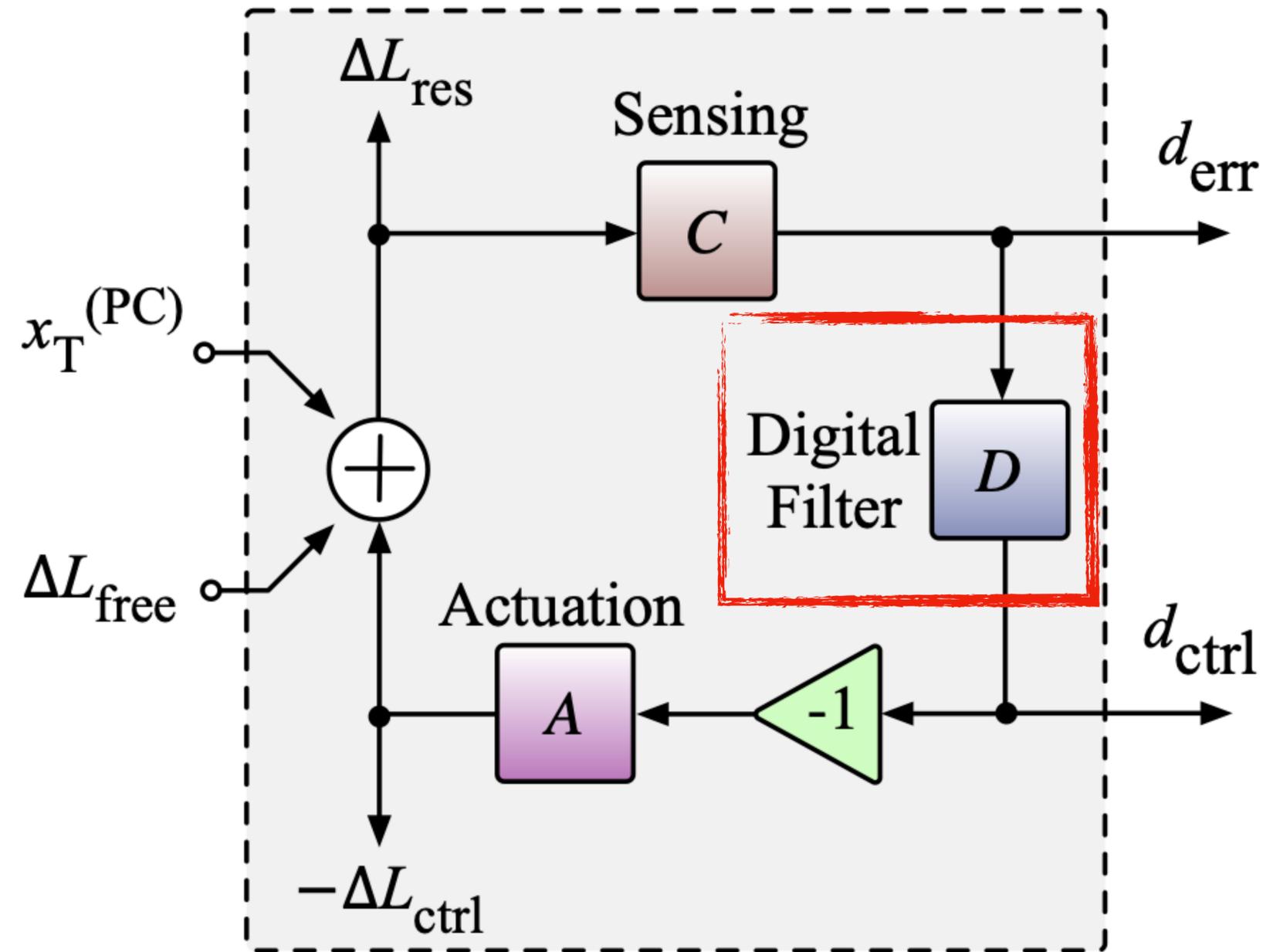
Frequency-domain response function:

$$\tilde{R}(f) = \frac{1}{\tilde{C}(f)} + \tilde{A}(f)\tilde{D}(f)$$

The feedback control loop (DARM loop)



The feedback control loop (DARM loop)

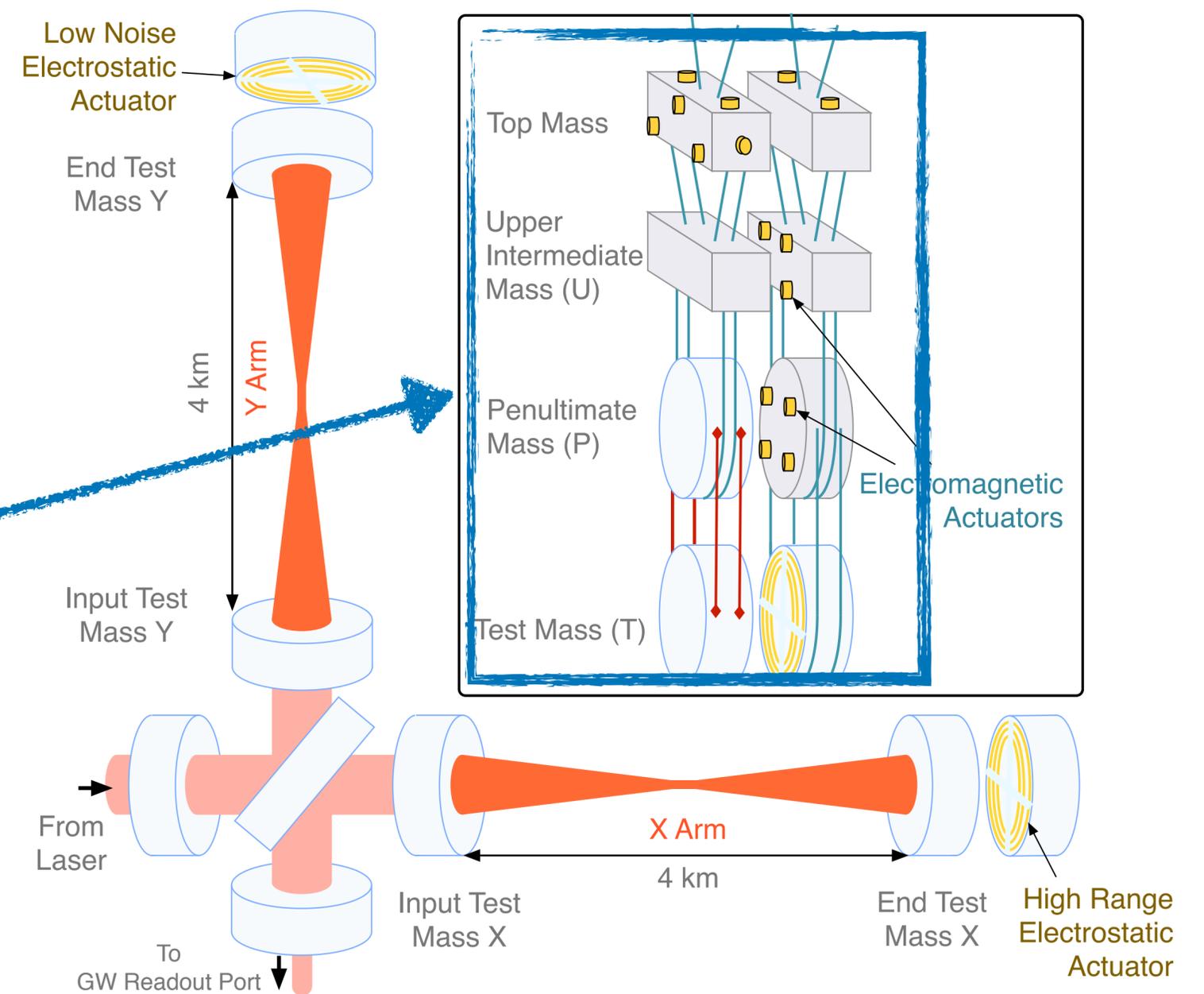
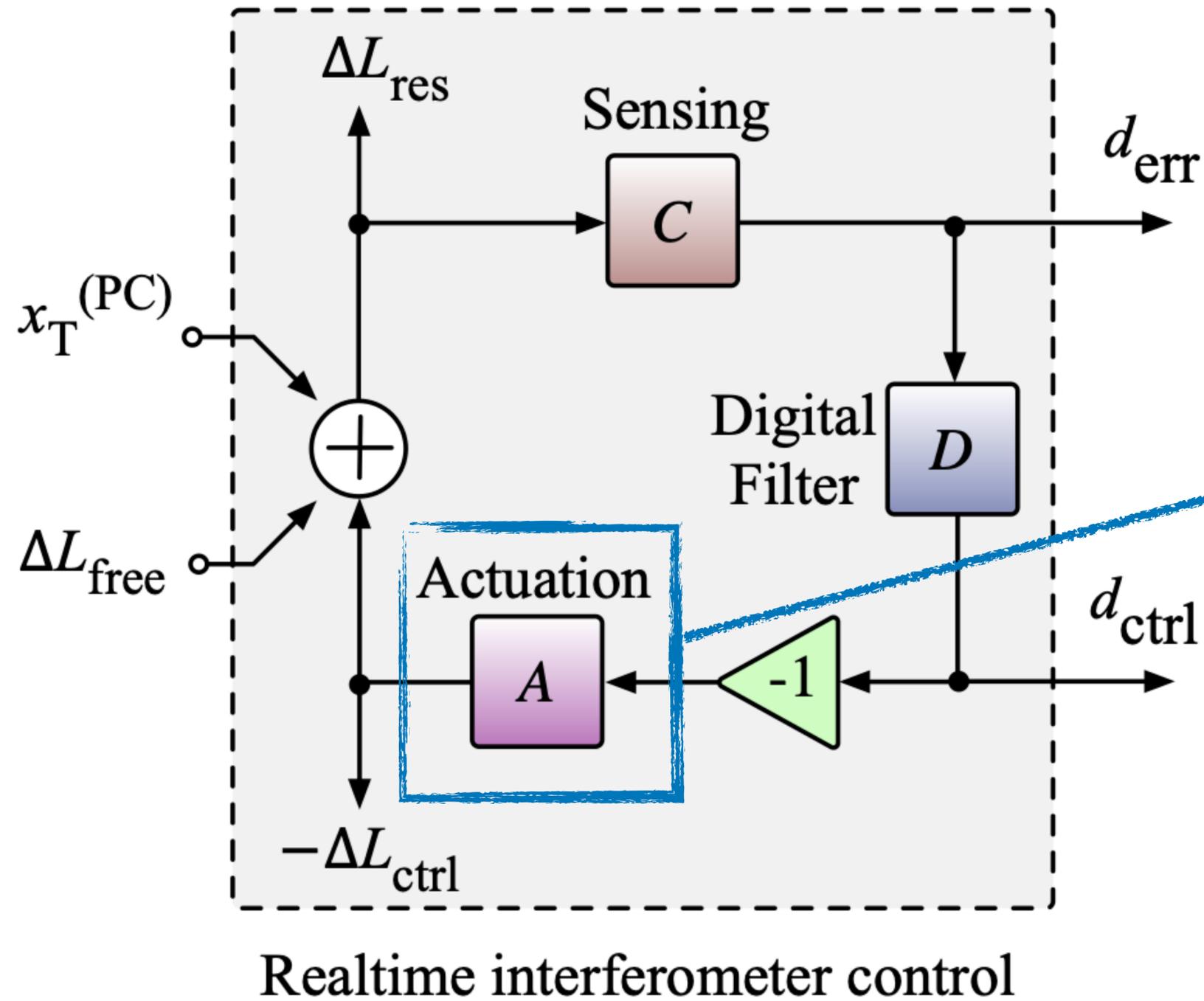


Realtime interferometer control

Low pass filters to isolate the lower frequency actuation

Notch filters to ensure that actuators are NOT actuating at the resonant frequencies

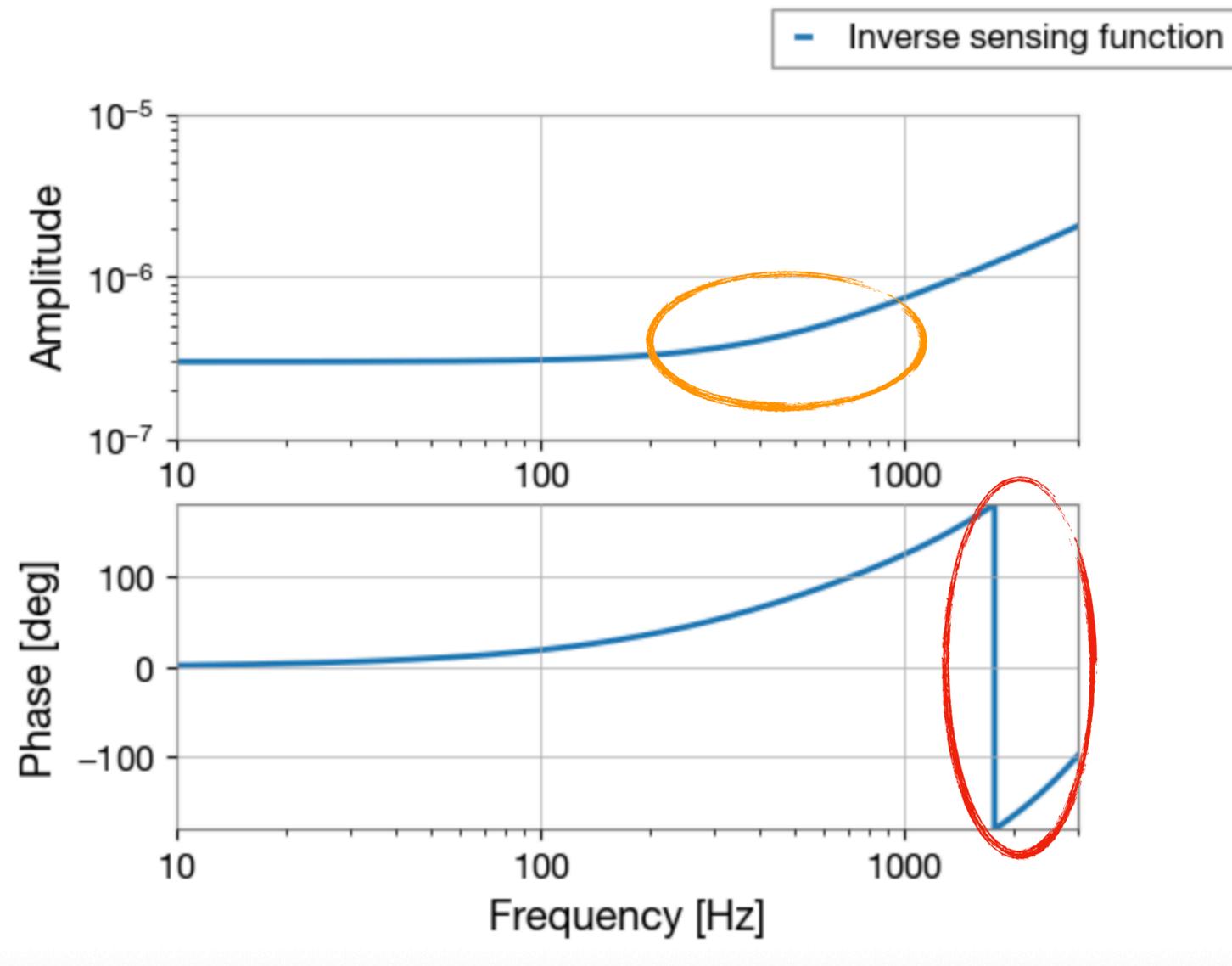
The feedback control loop (DARM loop)



Phys. Rev. D 96, 102001

Sensing function

- Transfer function from the differential arm (DARM) displacement to photodetector output current
- Below 1 kHz determined by Fabry-Perot cavity response and the signal recycling cavity response
- Above 1 kHz determined by ADC process

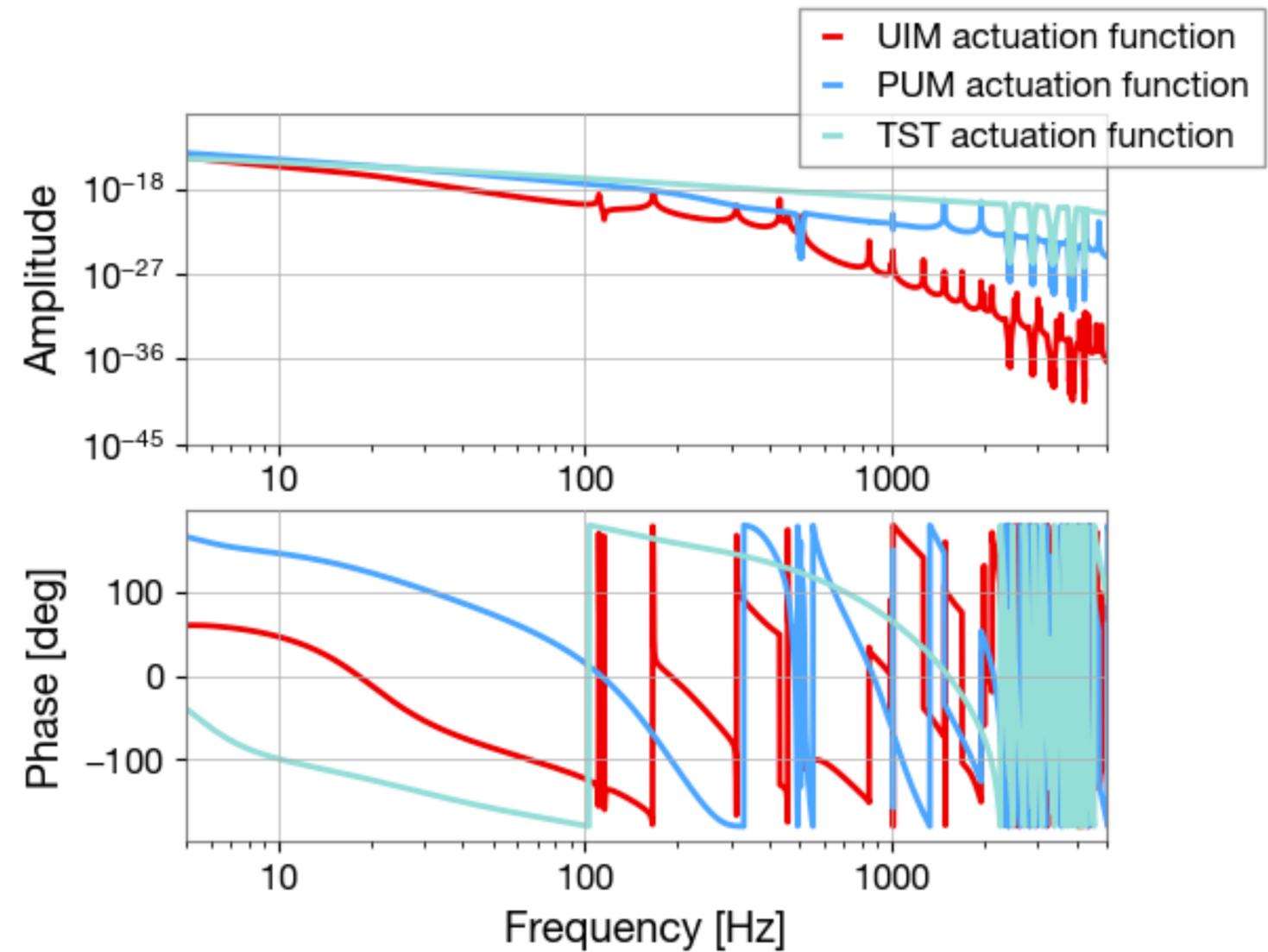
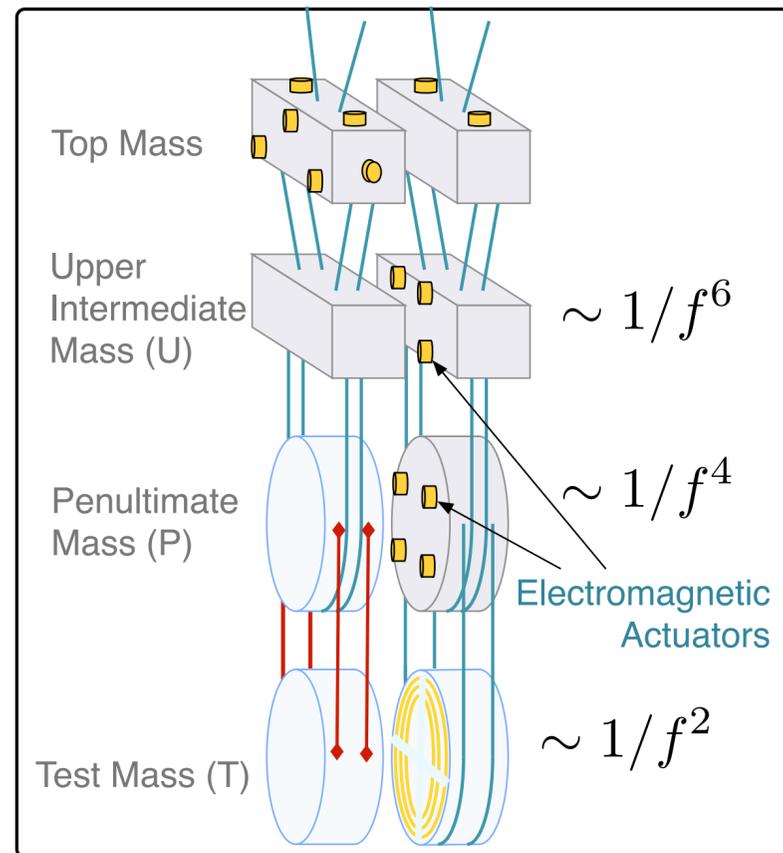


Let's look at some of the elements:

$$\tilde{C}(f; t) = \kappa_C(t) H_C \left(\frac{1}{1 + i f f_{cc}^{-1}(t)} \right) \left(\frac{f^2}{f^2 + f_s^2(t) - i f f_s(t) Q^{-1}(t)} \right) C_R(f) \exp(-2\pi i f \tau_C)$$

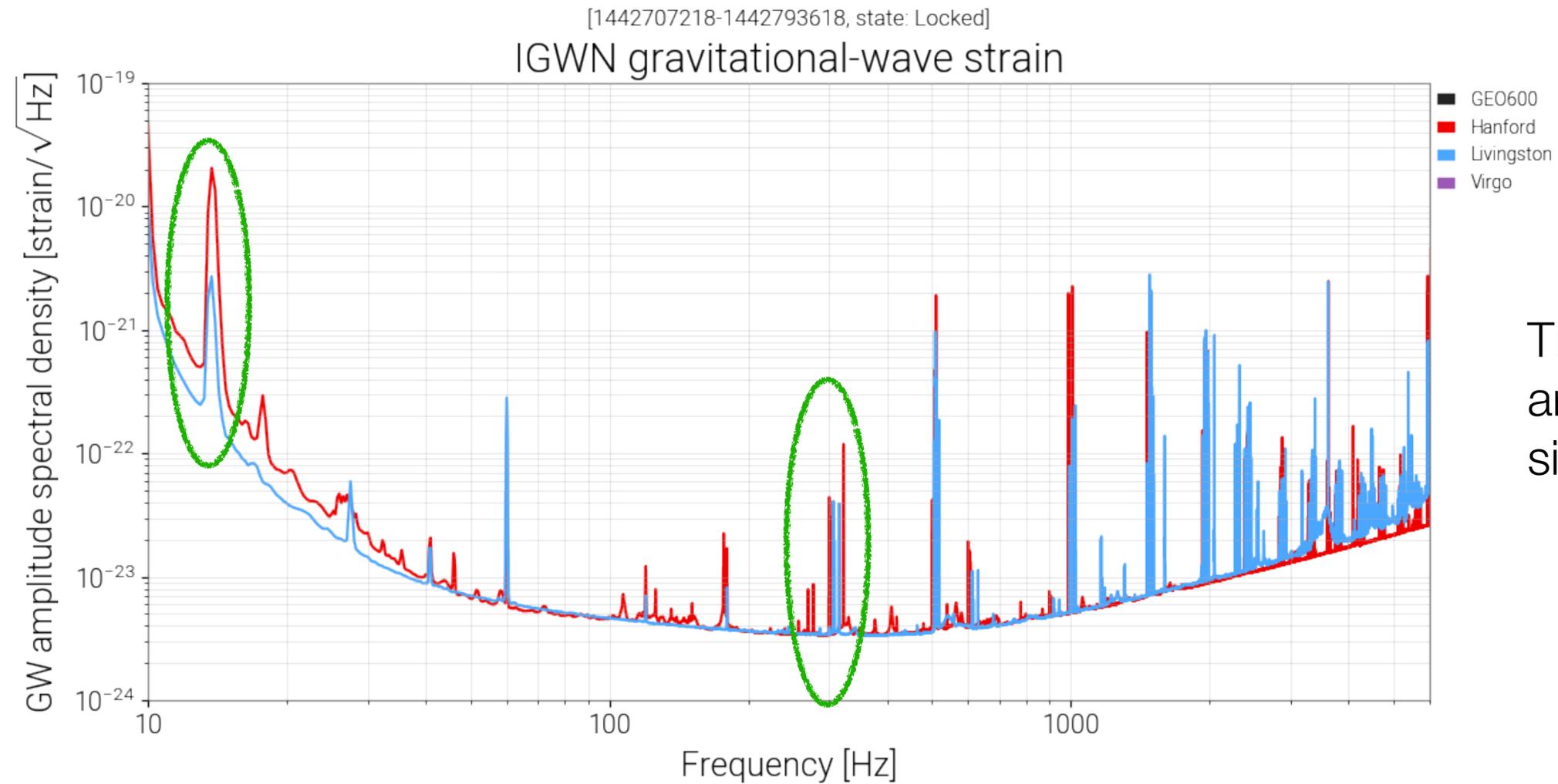
Actuation function

Mechanical response of each stage of pendulum



$$\tilde{A}(f; t) = \left[\kappa_U(t) \tilde{A}_U(f) + \kappa_P(t) \tilde{A}_P(f) + \kappa_T(t) \tilde{A}_T(f) \right] \exp(-2\pi i f \tau_A)$$

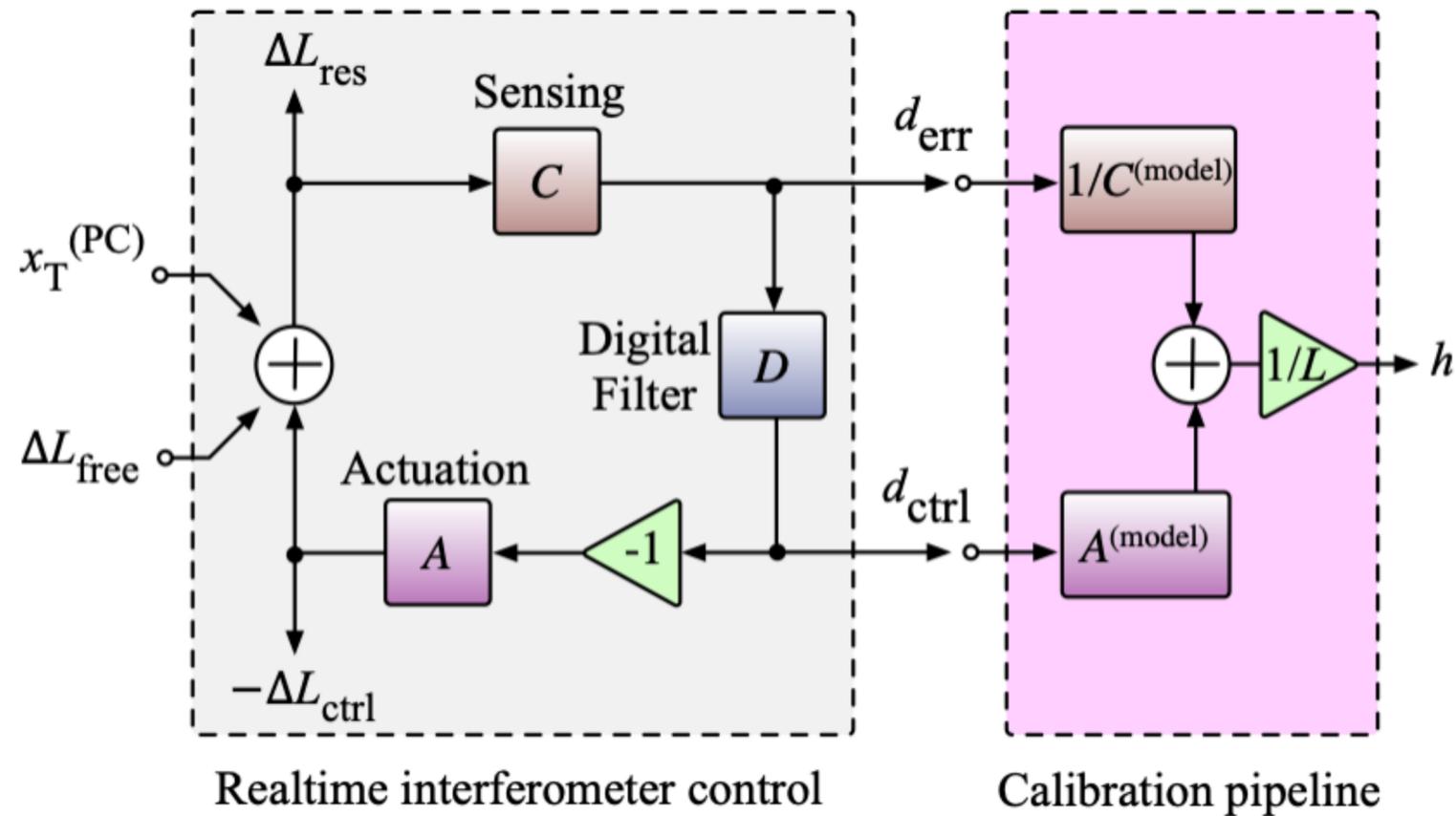
Calibration lines to track time-dependent parameter



Time-dependent parameters are tracked using narrow-band sinusoidal injections

Going back to DARM loop: reconstructing $h(t)$

Calibration involves inverting the effect of Sensing, Digital Filter and Actuation to reconstruct the timeseries for strain



$$h(t) L = \frac{1}{C} * d_{\text{err}} + A * d_{\text{ctrl}}$$

$$h(t) = R * d_{\text{err}}(t)$$

Frequency-domain response function:

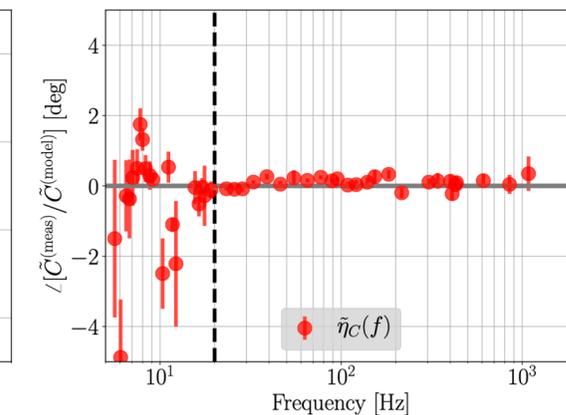
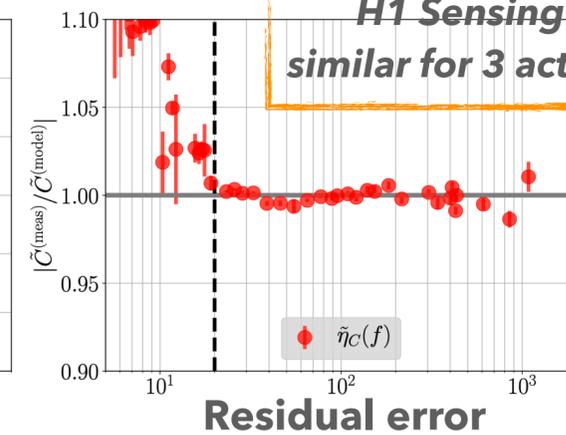
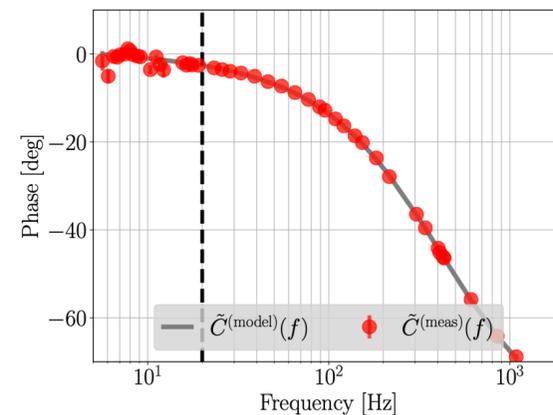
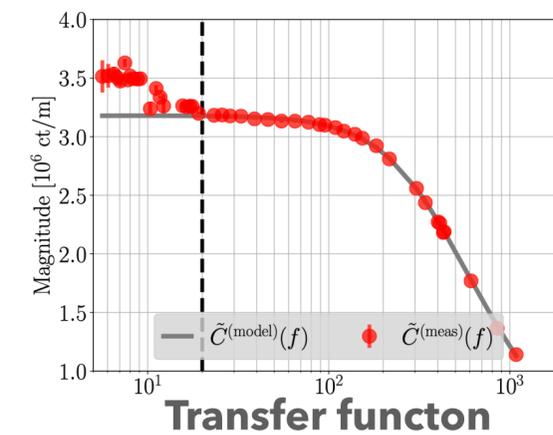
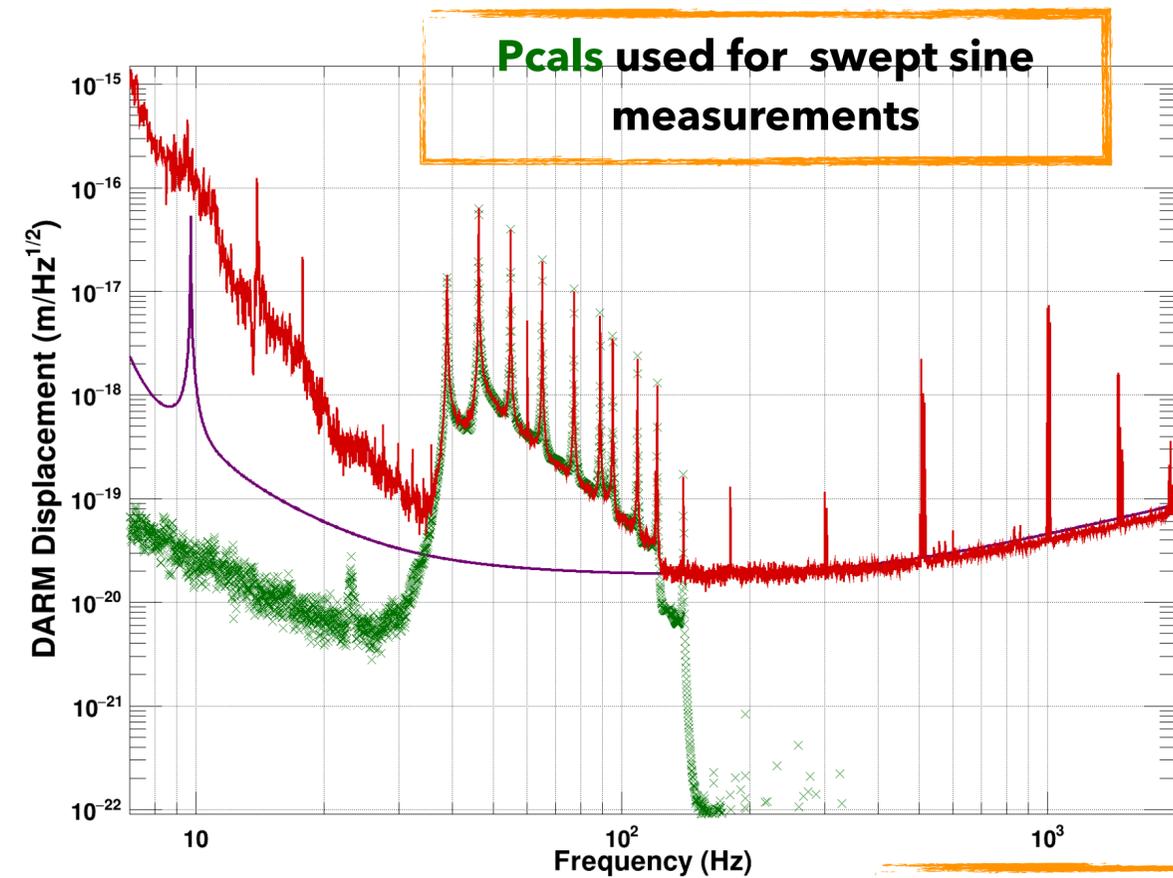
$$\tilde{R}(f) = \frac{1}{\tilde{C}(f)} + \tilde{A}(f)\tilde{D}(f)$$

Frequency dependent systematic error of the response function:

$$\eta_R = R^{(\text{meas})} / R^{(\text{model})}$$

Calibration process: broad steps

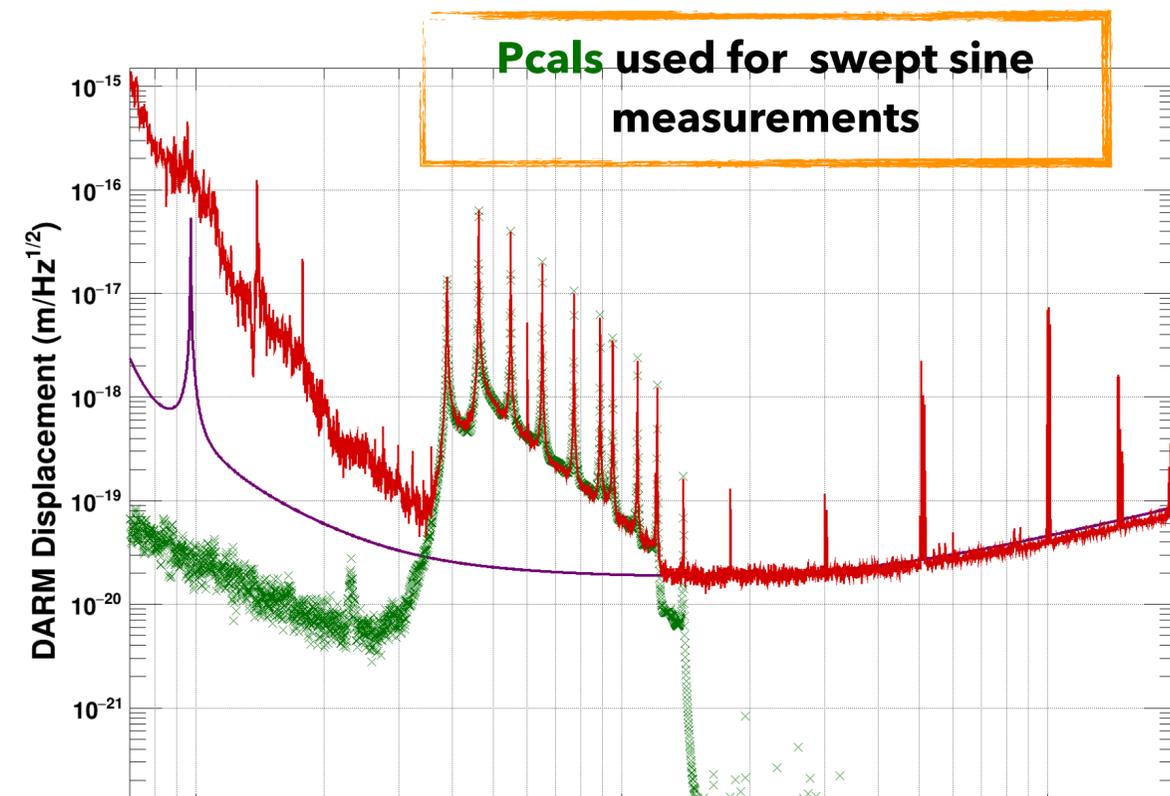
1. Before run starts Pcal is calibrated
2. Construct a static reference model of the response function by making interferometric measurements and estimating remaining physical parameters.
3. Take weekly “swept sine” measurements throughout the run to track time dependence of the parameters throughout the run
4. Compute residuals between the the model and measurements after correcting for time dependent variations
5. Construct probability distribution of η_R using GPR
6. The median represents systematic error



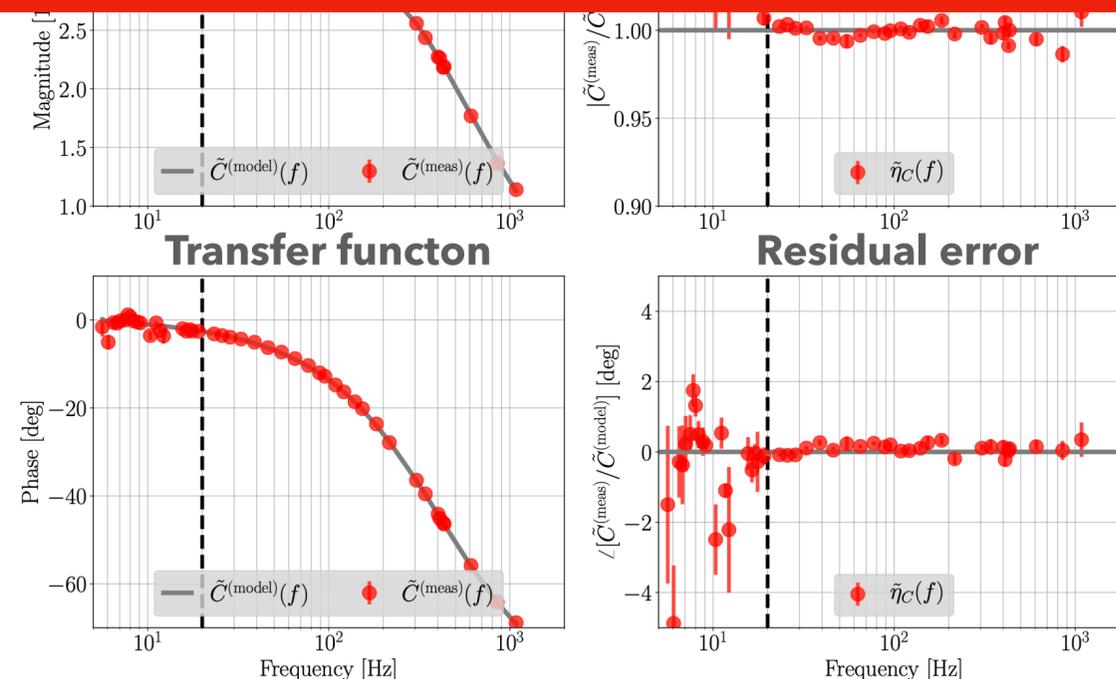
*H1 Sensing example;
similar for 3 actuation stages*

Calibration process: broad steps

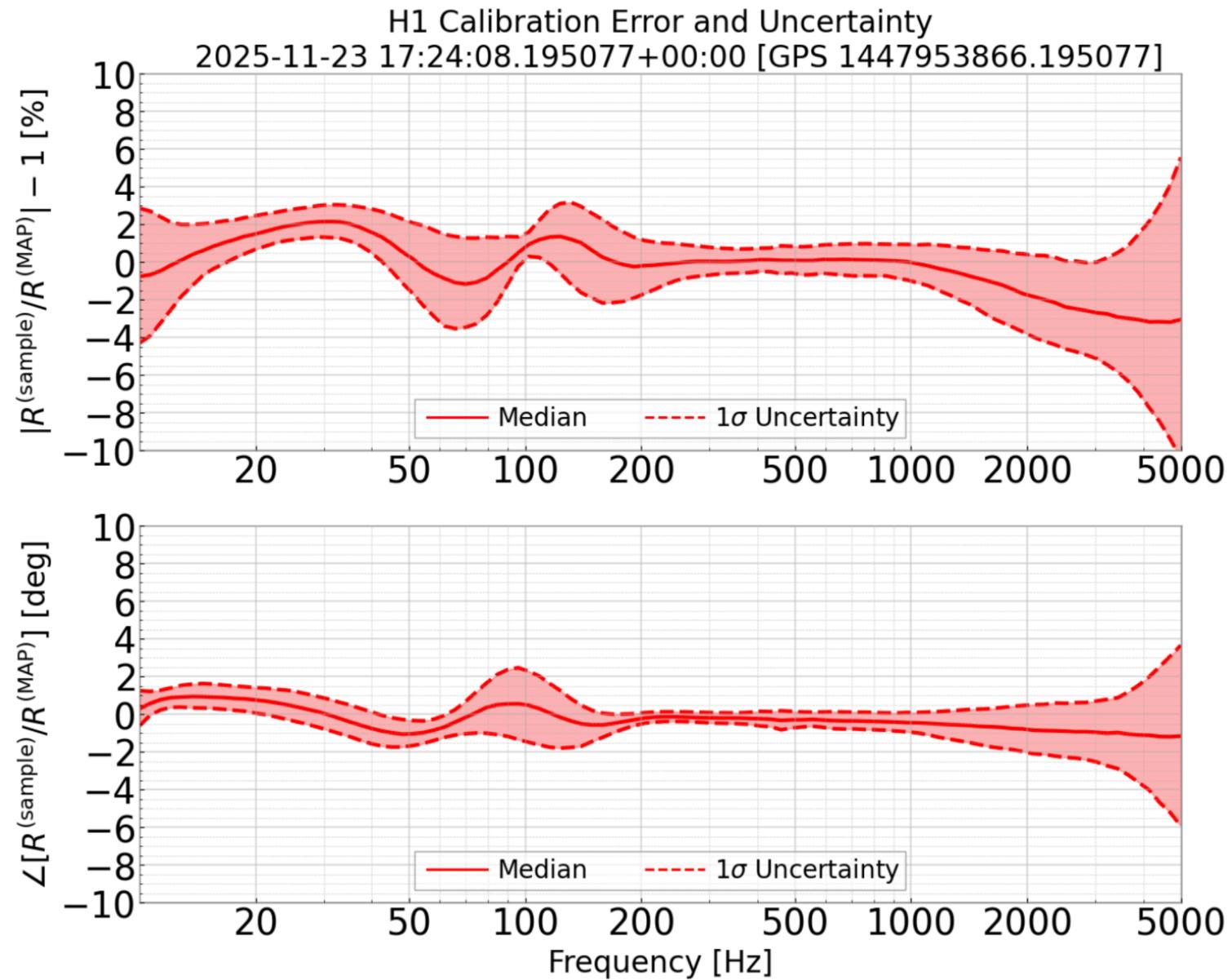
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4. Com
5. Construct probability distribution of η_R using GPR
6. The median represents systematic error



We will do a couple of these steps at the hands-on session tomorrow...



How accurately we reconstruct $h(t)$?



Current overall calibration systematic error is $\sim 2\%$ in the sensitive frequency band region, 20Hz - 2000Hz .

[L. Sun et.al Class. Quantum Grav. 37 225008 \(2020\)](#)

Sufficiently small for astrophysical parameter estimation

[Vitale et. al arXiv:2009.10192 \(2020\).](#)

[Payne et. al Phys Rev D. 102.12 \(2020\): 122004](#)

Questions?