IUCAA workshop
Day 1: Optics

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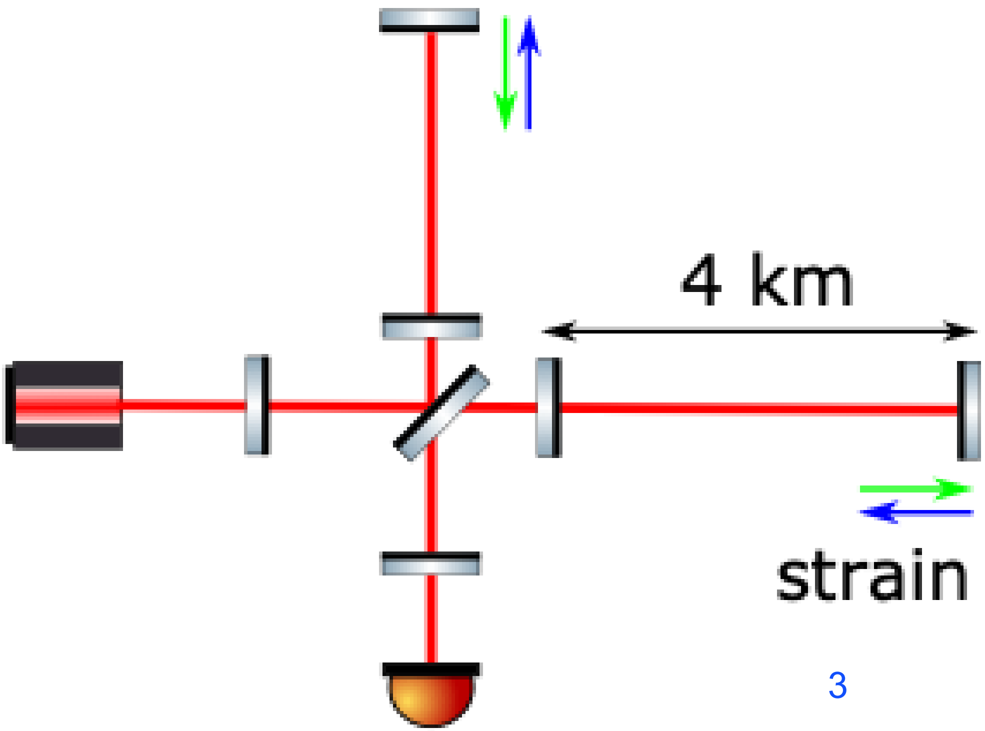
The programme

- Day 1: Interferometry and feedback control
- Day 2: Mechatronics
- Day 3: Cavity locking
- Day 4: Angular controls and noises
- Day 5: Practical issues and solutions

Overview

- Interference of light
- Michelson interferometer
- Fabry Perot interferometer
- Laser beams



Wave equation and solutions

- Maxwell's wave equation in vacuum

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0$$

- Plane waves

$$\mathbf{E}(\mathbf{r}, t) = E_0 e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})} + c.c.$$

- Gaussian beams
- Spherical waves
- Waveguide modes

Examples of light applications

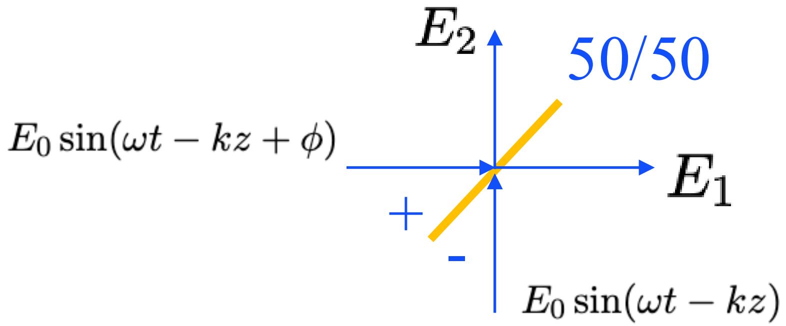
Plane wave

$$\mathbf{E}(\mathbf{r},t) = E_0 e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})} + c.c.$$

- Amplitude quadrate
  - Trapping
  - Cutting
- Phase quadrature
  - Precision measurements
- Entanglement
  - Optical squeezing

How to measure phase?

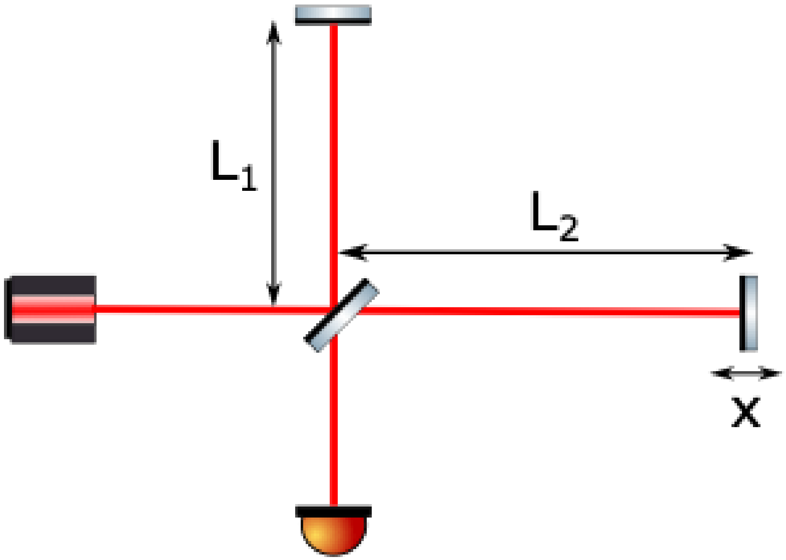
- Electrons in a photodetector are too slow to measure the phase of light
- We need an interference of two waves



$$E_1 = \frac{E_0}{\sqrt{2}}\big(\sin(\omega t - kz + \phi) - \sin(\omega t - kz)\big) = \sqrt{2}E_0 \sin\frac{\phi}{2} \cos\left(\omega t - kz + \frac{\phi}{2}\right)$$

$$E_2 = \frac{E_0}{\sqrt{2}}\big(\sin(\omega t - kz + \phi) + \sin(\omega t - kz)\big) = \sqrt{2}E_0 \cos\frac{\phi}{2} \sin\left(\omega t - kz + \frac{\phi}{2}\right)$$

Michelson interferometer



Electric field at the antisymmetric port

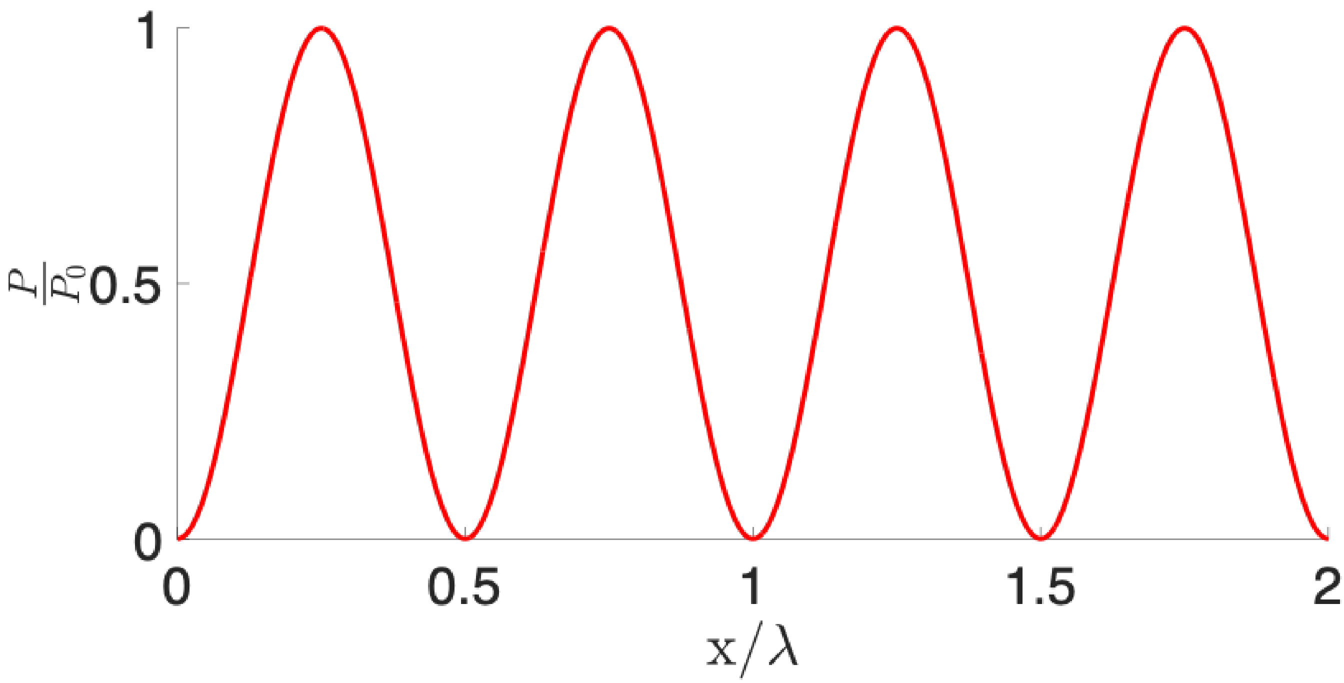
$$E = \frac{1}{2} E_0 \left(\sin(\omega_0 t - 2kL_1) - \sin(\omega_0 t - 2kL_2)\right) = E_0 \sin\left(\frac{2\pi \Delta L}{\lambda}\right) \cos\left(\omega_0 t - \frac{2\pi (L_1 + L_2)}{\lambda}\right)$$

Power at the antisymmetric port

$$P = P_0 \sin^2\left(\frac{2\pi \Delta L}{\lambda}\right) = \frac{P_0}{2}\left(1 - \cos\frac{4\pi \Delta L}{\lambda}\right)$$

Michelson interferometer

For convenience, we set $L_x - L_y = N * \text{lambda} + x$.



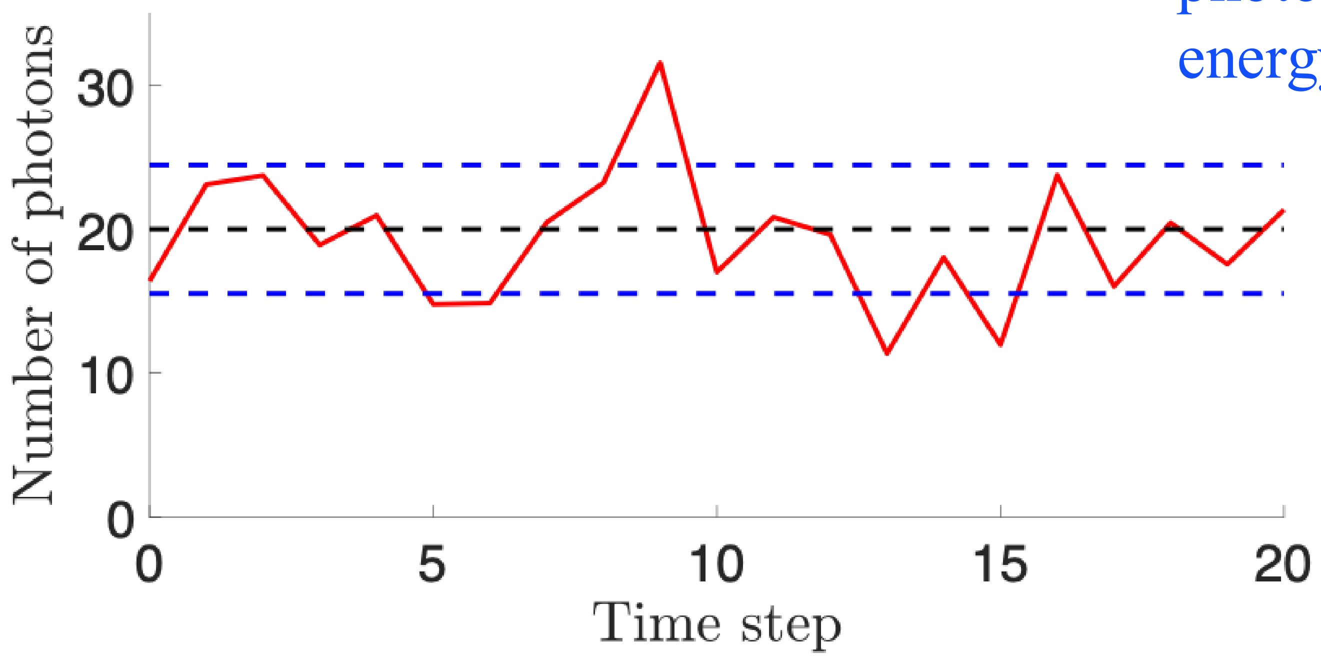
The power swings from min to max if the target mirror moves by 266 nm.

Shot noise

- The power fluctuates even if the sample mirror does not move.
- The average number of photons $\langle N \rangle = \frac{P\tau}{h\nu},$
- The standard deviation is $\sqrt{\langle N \rangle}.$

measurement time

photon energy



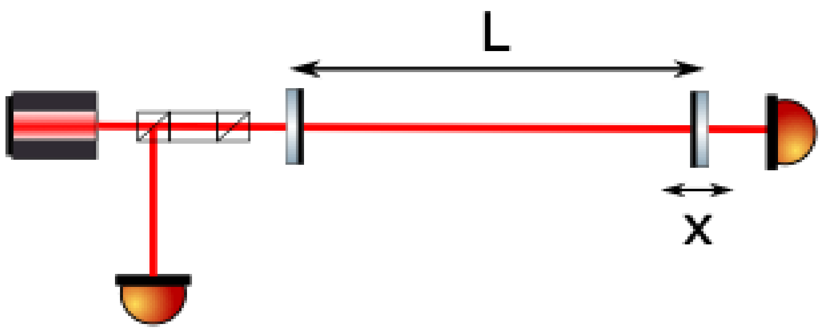
Quantum-limited resolution

- Find the optical gain, the derivative of the measured power over the mirror motion, x, at the operational point in units of W/m.
- Find the standard deviation of the random power fluctuations in units of W.
- Divide the shot noise by the optical gain to find the quantum-limited resolution in units of m.

Exercise 1

Calculate the quantum-limited resolution of a Michelson interferometer with the following parameters: operating point x_0 = 0.03 lambda, lambda = 1064 nm, measurement time tau = 1 msec, and the input laser power is 1 W.

Fabry-Perot interferometer



Lossless mirrors:

$$r^2 + t^2 = 1$$

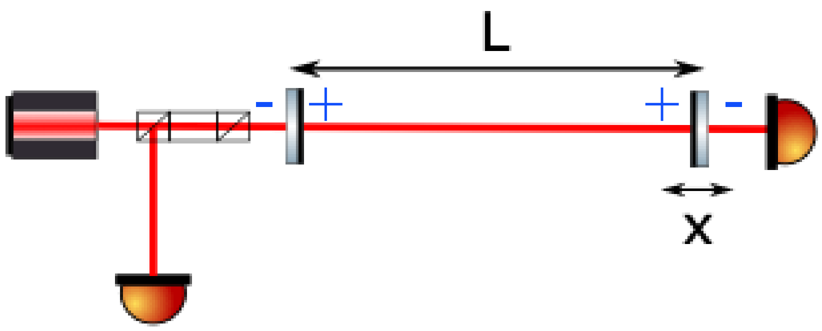
Complex fields

$$E(z=0) = E_0 \sin(\omega_0 t + \phi_0) = \frac{E_0 e^{i\phi_0}}{2i} e^{i\omega_0 t} + c.c. = E_1 e^{i\omega_0 t} + c.c.,$$

Free space field propagation

$$E_2 = E_1 e^{-i\phi}, \qquad \phi = kz$$

Fabry-Perot interferometer



Interference condition at the input mirror

$$E_{\text{cav}} = E_{\text{in}} t + r^2 E_{\text{cav}} e^{-i\phi_{rt}}, \qquad \phi_{rt} = 2kL$$

Solution to the intracavity field:

$$E_{\text{cav}} = \frac{t}{1 - r^2 e^{-i\phi_{rt}}} E_{\text{in}}$$

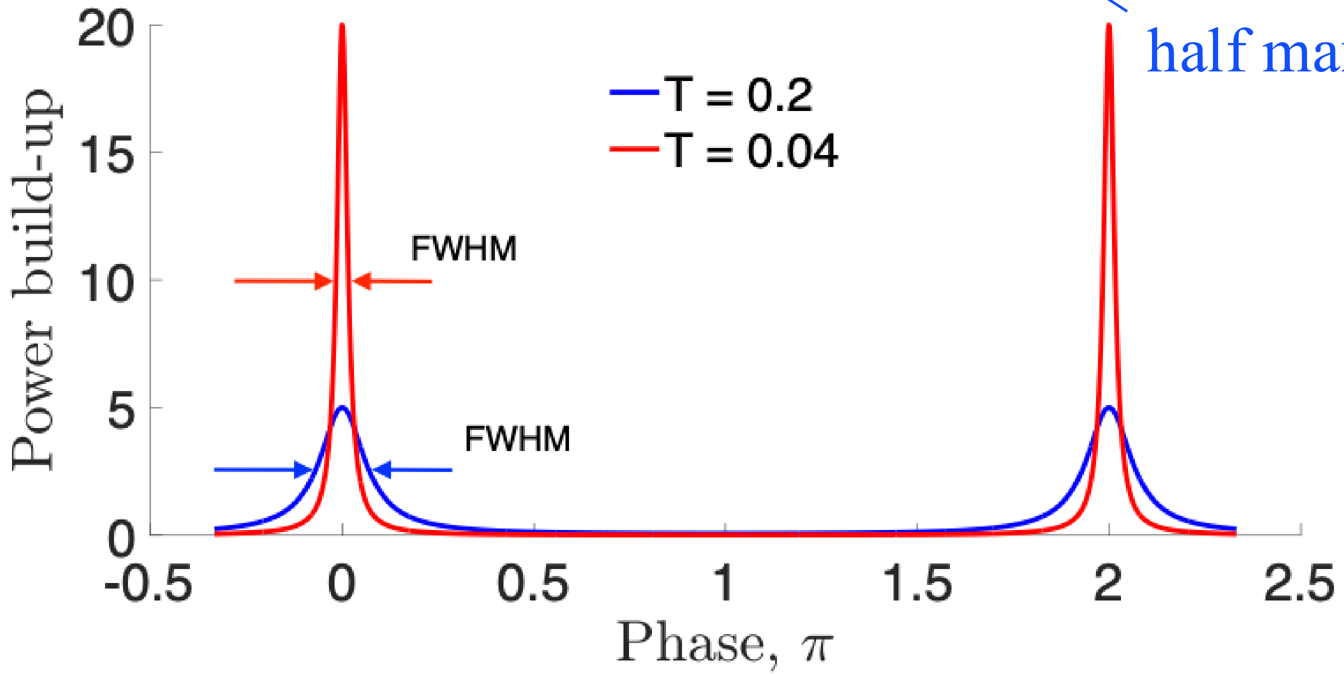
Fabry-Perot interferometer

- Intracavity power $$P_{\text{cav}} = \frac{T}{|1 - r^2 e^{-i\phi_{rt}}|^2} P_{\text{in}}.$$

$T = t^2$
$R = r^2$

- Define the optical finesse as $$\mathcal{F} = \frac{2\pi}{\Delta\phi_{rt}}$$

Full width at half max (FWHM)



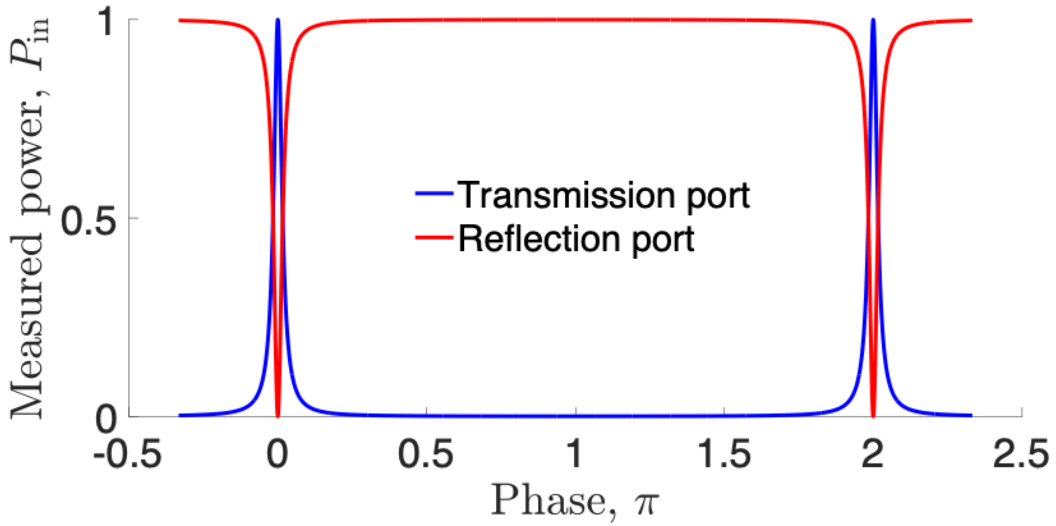
Fabry-Perot interferometer

- Transmission port

$$P_{\text{tr}} = \frac{T^2}{|1 - r^2 e^{-i\phi_{rt}}|^2} P_{\text{in}}$$

- Reflection port

$$E_{\text{refl}} = -rE_{\text{in}} + trE_{\text{cav}} e^{-i\phi_{rt}} \qquad P_{\text{refl}} = \left| \frac{-r + re^{-i\phi_{rt}}}{1 - r^2 e^{-i\phi_{rt}}} \right|^2 P_{\text{in}}.$$



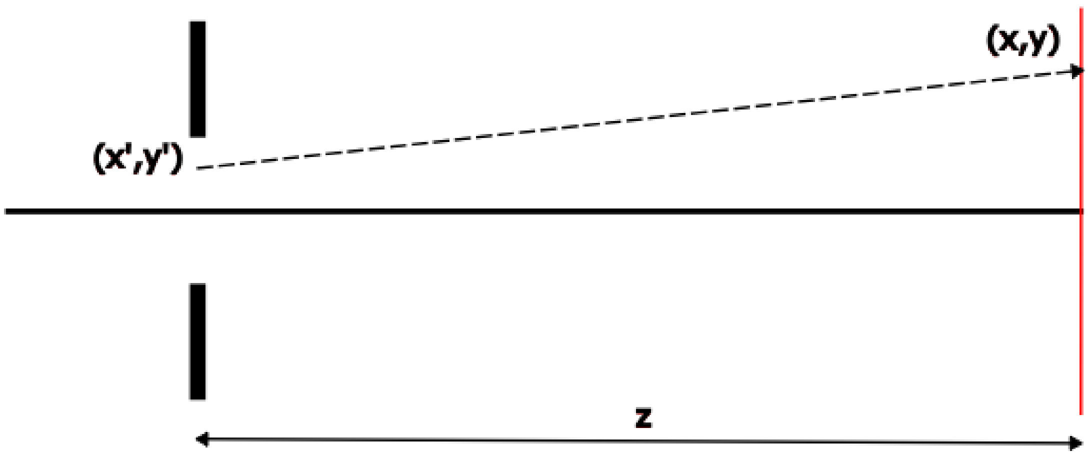
Exercise 2

a) Find the maximum and minimum power amplification factors in a resonator with two identical mirrors with power transmissivity of $T=10^{-5}$.

b) Estimate the shot noise limited resolution of a Fabry-Perot interferometer with identical mirrors of power transmissivity of $T=10^{-5}$. Input laser power is 1 W, wavelength is 1064 nm, measurement time is 1 msec. The measurement is done via the reflection port.

Gaussian beams

Fraunhofer diffraction: powerful result from the Fourier Optics



$$E(x,y,z) \sim \int_S E(x',y') \exp(i\frac{k}{z}(xx' + yy'))dx'dy'$$

If the beam preserves its shape, then the shape should be an eigen function of the Fourier Transform.

Wave equation

We search for the solutions to the Maxwell's wave equation

$$\frac{\partial^2 \mathbf{E}}{c^2 \partial t^2} - \left(\frac{\partial^2 \mathbf{E}}{\partial x^2} + \frac{\partial^2 \mathbf{E}}{\partial y^2} + \frac{\partial^2 \mathbf{E}}{\partial z^2} \right) = 0$$

in the form

$$\mathbf{E}(x, y, z, t) = \mathbf{e_x} E(x, y, z) \exp(i\omega t - ikz),$$

with an assumption that E is a slow function of z: $\frac{\partial E}{\partial z} \ll kE.$

We arrive to the equation

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} = 2ik \frac{\partial E}{\partial z};$$

Solution to the wave equation

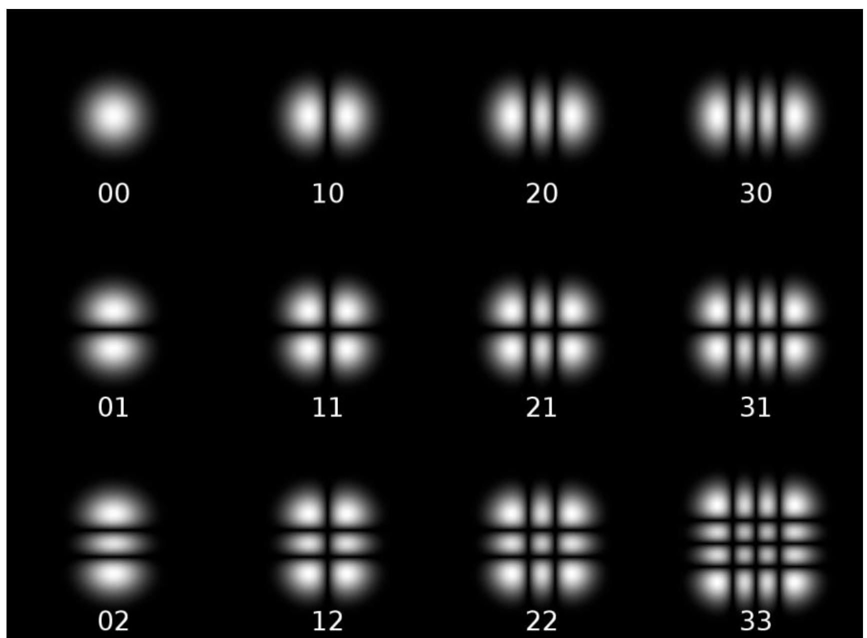
$$E_{lm}(x,y,z) = E_0 \frac{w_0}{w(z)} H_l\left(\frac{\sqrt{2}x}{w(z)}\right) H_m\left(\frac{\sqrt{2}y}{w(z)}\right) \exp\left(-(x^2+y^2)\left(\frac{1}{w(z)^2} + \frac{ik}{2R(z)}\right)\right) \exp(i\psi(z))$$

Hermite polynomials

Beam size

Radius of curvature

Gouy phase



Beam propagation: ABCD matrices

- Introduce a q-parameter:

$$\frac{1}{q} = \frac{1}{R} - \frac{i\lambda}{\pi w^2}.$$

- Propagation of the q-parameter:

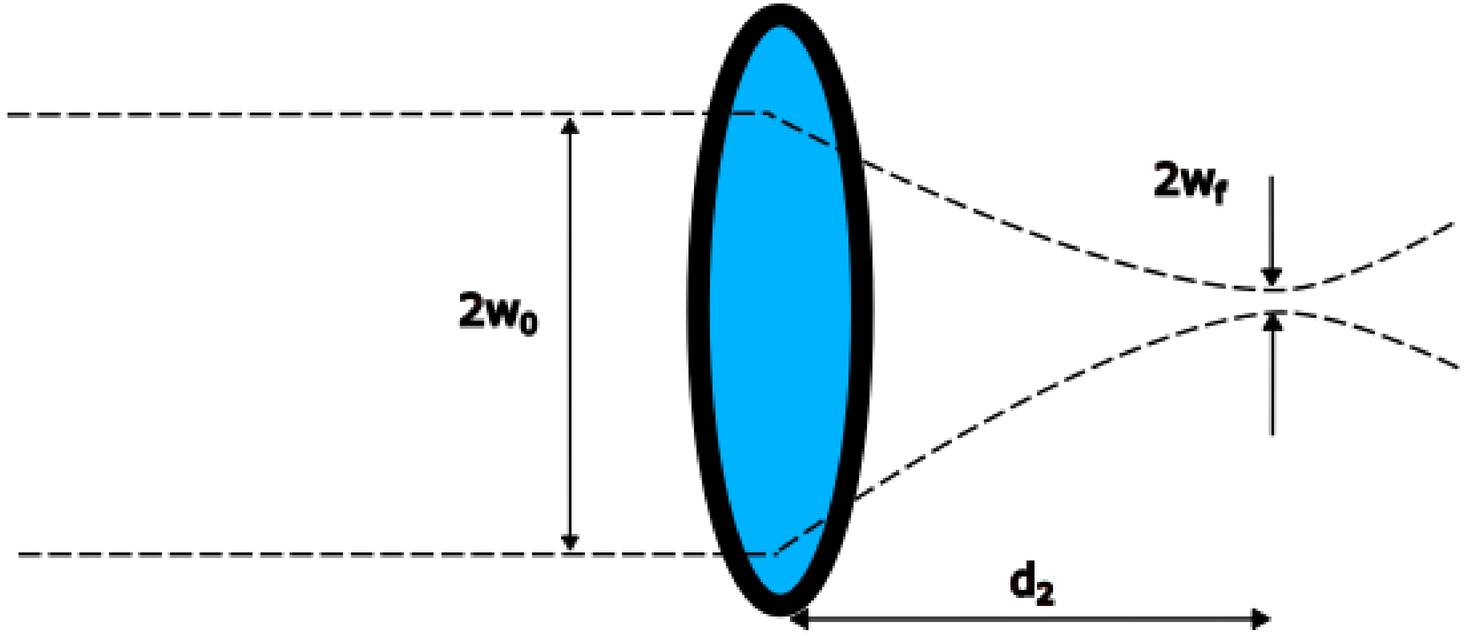
$$\begin{pmatrix} q_2 \\ 1 \end{pmatrix} = kM \begin{pmatrix} q_1 \\ 1 \end{pmatrix}$$

- Example: free space propagation

$$\begin{pmatrix} q_2 \\ 1 \end{pmatrix} = k \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} q_1 \\ 1 \end{pmatrix} \longrightarrow \begin{aligned} k &= 1 \\ q_2 &= q_1 + L \end{aligned}$$

Exercise 3

Find d_2 and w_f assuming z_0 = pi w_0^2 / lambda >> f.



Cavity lab

Study experimental aspects of an optical cavity

- » Polarization filtering
- » Mode matching
- » Alignment
- » Mode resonances
- » Beam parameters