IUCAA workshop
Day 3: Cavities

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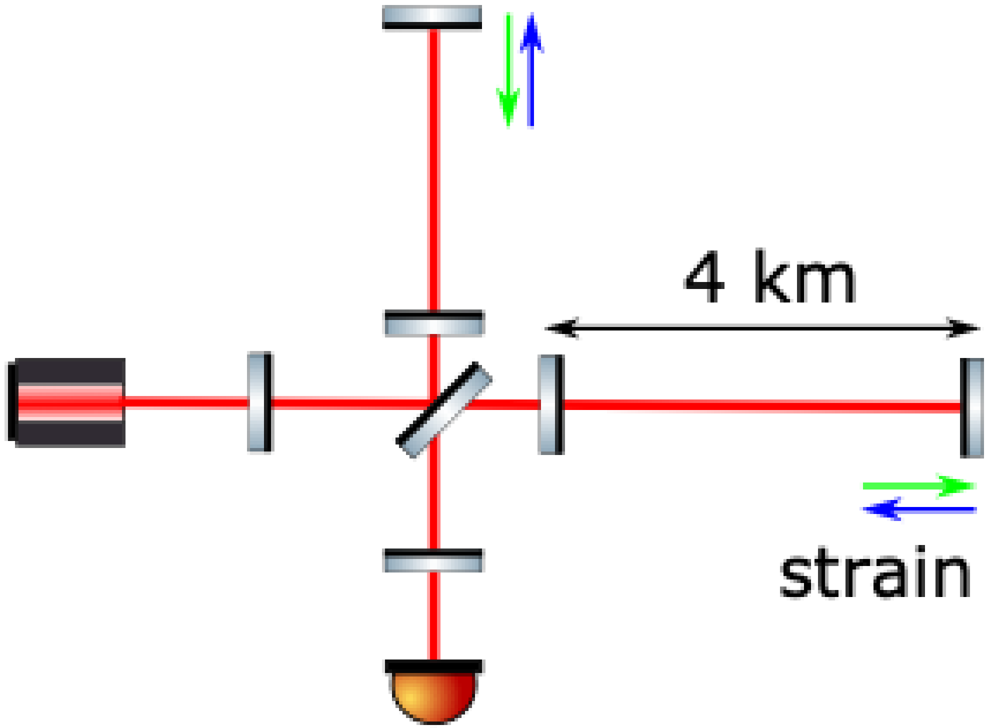
The programme

- Day 1: Interferometry and feedback control
- Day 2: Mechatronics
- Day 3: Cavity locking
- Day 4: Angular controls and noises
- Day 5: Practical issues and solutions

Overview

- Gaussian modes
- Frequency response of optical cavities
- Lock acquisition strategy



Recap: wave equation

We search for the solutions to the Maxwell's wave equation

$$\frac{\partial^2 \mathbf{E}}{c^2 \partial t^2} - \left(\frac{\partial^2 \mathbf{E}}{\partial x^2} + \frac{\partial^2 \mathbf{E}}{\partial y^2} + \frac{\partial^2 \mathbf{E}}{\partial z^2} \right) = 0$$

in the form

$$\mathbf{E}(x, y, z, t) = \mathbf{e_x} E(x, y, z) \exp(i\omega t - ikz),$$

with an assumption that E is a slow function of z: $\frac{\partial E}{\partial z} \ll kE.$

We arrive to the equation

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} = 2ik \frac{\partial E}{\partial z}$$

Recap: Solution to the wave equation

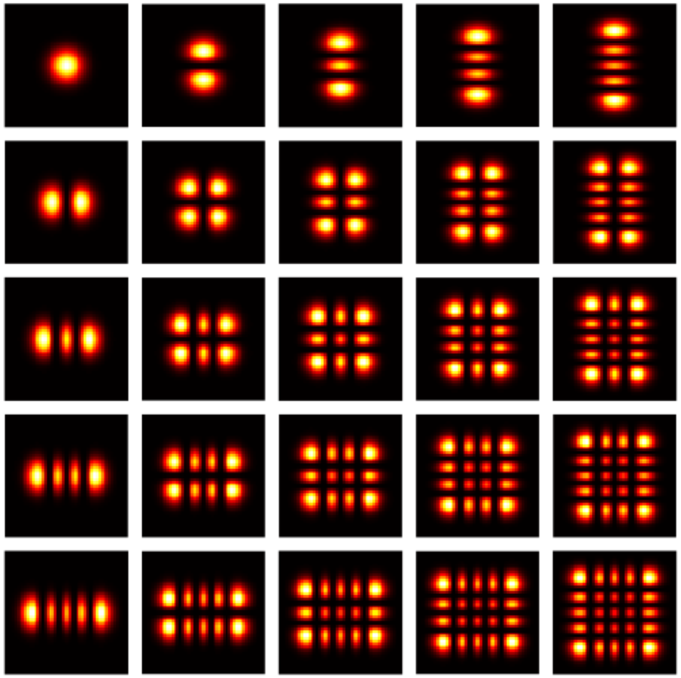
$$E_{lm}(x,y,z) = E_0 \frac{w_0}{w(z)} H_l\left(\frac{\sqrt{2}x}{w(z)}\right) H_m\left(\frac{\sqrt{2}y}{w(z)}\right) \exp\left(-(x^2+y^2)\left(\frac{1}{w(z)^2} + \frac{ik}{2R(z)}\right)\right) \exp(i\psi(z))$$

Hermite polynomials

Beam size

Radius of curvature

Gouy phase

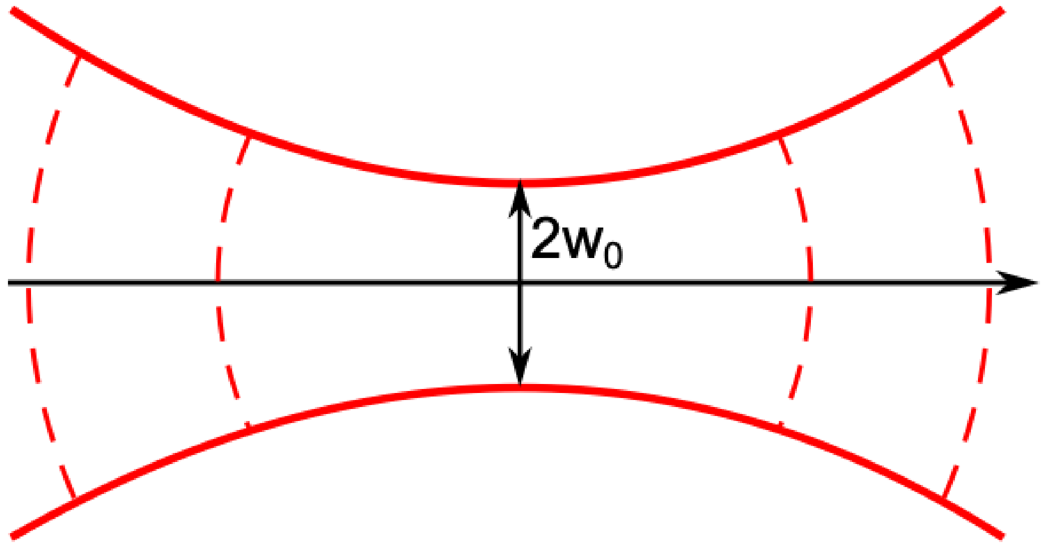


Beam propagation from the waist

Beam waist, radius of curvature, Gouy phase:

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2} \qquad R(z) = z + \frac{z_R^2}{z} \qquad \psi(z) = \arctan\left(\frac{z}{z_R}\right).$$

Rayleigh range: $z_R = \pi n w_0^2 / \lambda$



Total round trip phase

- Free space longitudinal phase
- Reflection phase
- Gouy phase
- Geometrical phase

- The resonance condition is

$$\phi_{rt} = kL + \phi_{\text{refl}} + \psi + \phi_g = 2\pi N$$

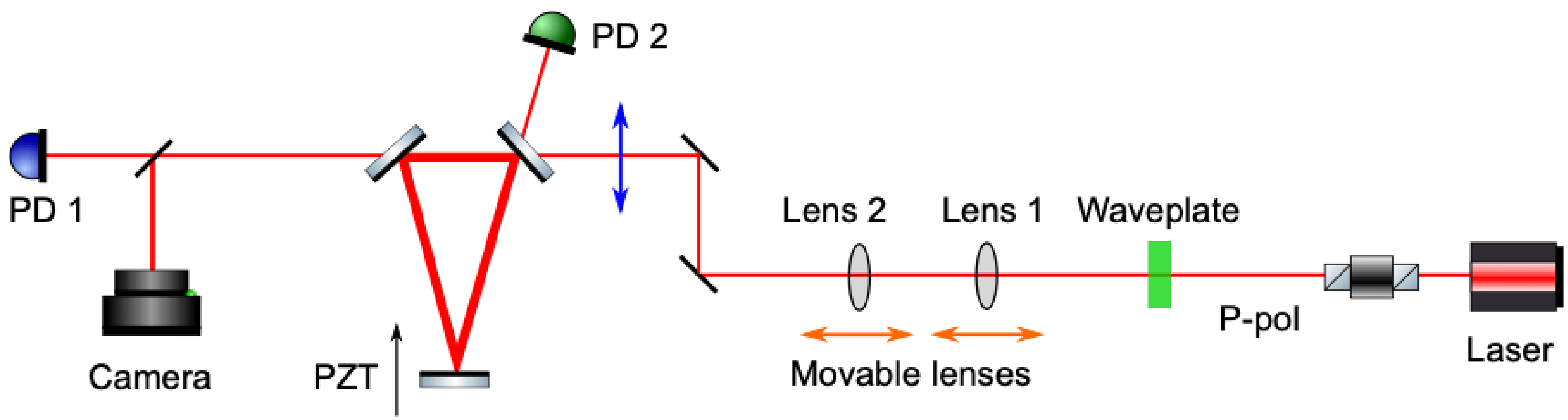
$$P_{\text{cav}} = \frac{T}{|1 - r^2 e^{-i\phi_{rt}}|^2} P_{\text{in}}.$$

Exercise 1

virtlab.quantumoptics.org.uk:5006/cavity

Study experimental aspects of an optical cavity

- » Mode resonances
- » Beam parameters



Recap: Beam propagation: ABCD matrices

- Introduce a q-parameter:

$$\frac{1}{q} = \frac{1}{R} - \frac{i\lambda}{\pi w^2}.$$

- Propagation of the q-parameter:

$$\begin{pmatrix} q_2 \\ 1 \end{pmatrix} = kM \begin{pmatrix} q_1 \\ 1 \end{pmatrix}$$

- Example: free space propagation

$$\begin{pmatrix} q_2 \\ 1 \end{pmatrix} = k \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} q_1 \\ 1 \end{pmatrix} \longrightarrow \begin{aligned} k &= 1 \\ q_2 &= q_1 + L \end{aligned}$$

Finding cavity modes

- If we propagate the beam q-parameter around the cavity, we should get the same q-parameter.

$$\begin{pmatrix} q_1 \\ 1 \end{pmatrix} = kM \begin{pmatrix} q_1 \\ 1 \end{pmatrix}$$

- May use this technique to find the q-parameter and the beam parameters.
- The mode matching between two coalined laser beams is given by the equation

$$\eta = \frac{4}{\left| \frac{q_1}{q_2} + \frac{q_2}{q_1} \right|^2}$$

Exercise 2

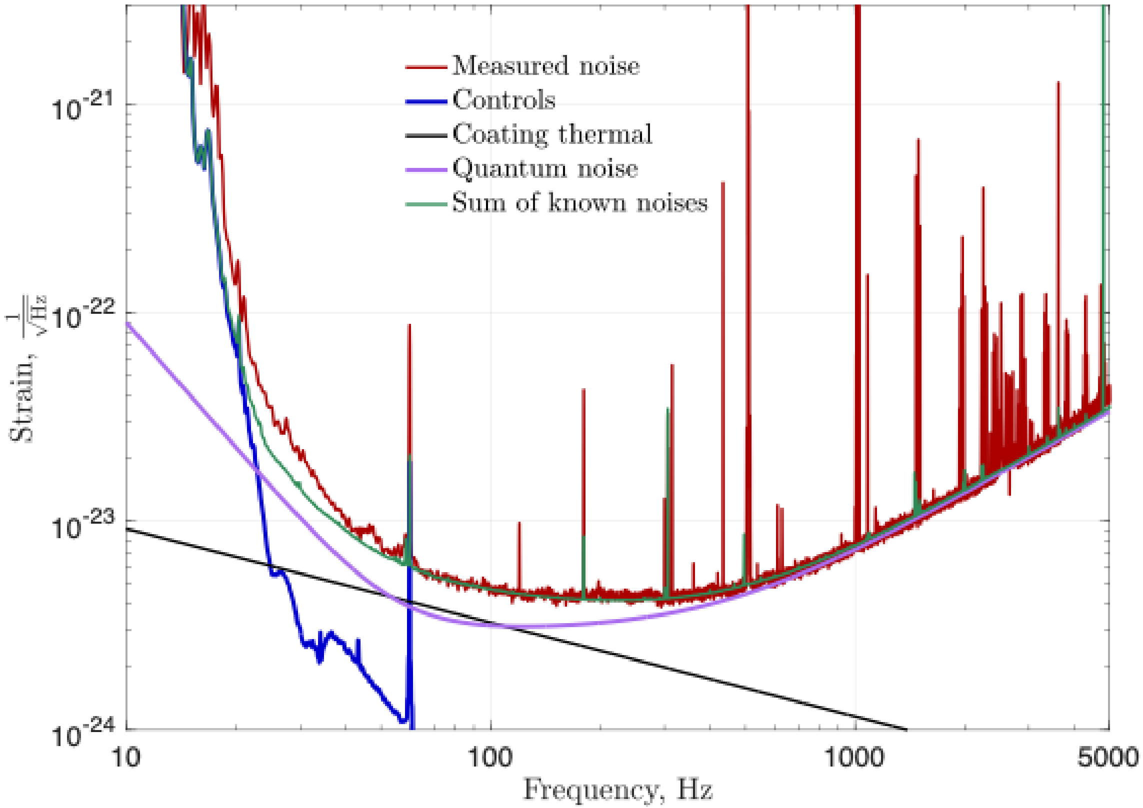
The LIGO arm is 3994.5 m long. The radii of curvature of the input and end test masses are 1934 m and 2245 m. The laser wavelength is 1064 nm. Find the beam size, w, on the input test mass.

Exercise 3*

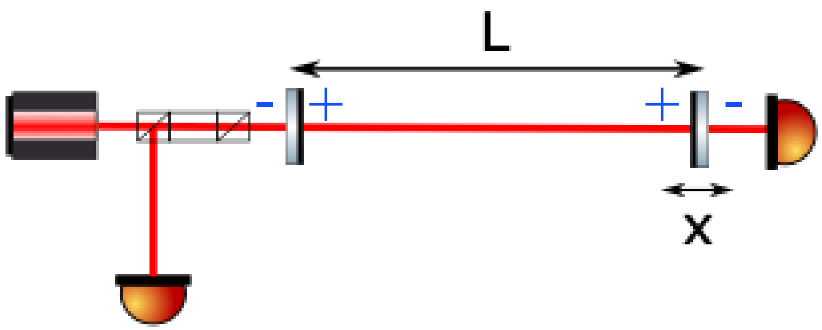
Compute the q-parameters of the LIGO power and signal recycling cavities and, using the result from Ex 2, find the mode matching between the arm and recycling cavities.

Frequency response of optical cavities

Similar to mechanical oscillators, the response depends on the excitation frequency.



Recap: Fabry-Perot interferometer



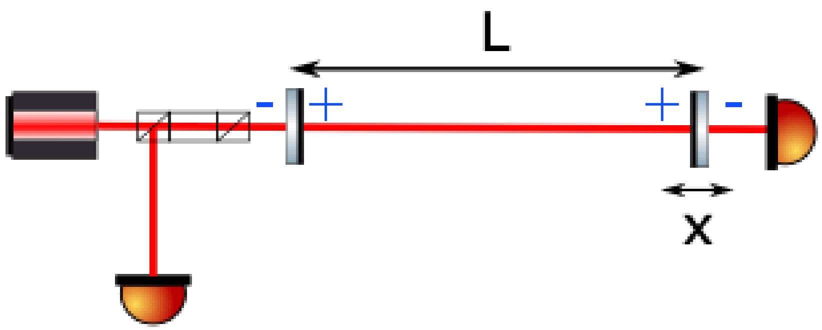
Interference condition at the input mirror

$$E_{\text{cav}} = E_{\text{in}} t + r^2 E_{\text{cav}} e^{-i\phi_{rt}}, \qquad \phi_{rt} = 2kL$$

Solution to the intracavity field:

$$E_{\text{cav}} = \frac{t}{1 - r^2 e^{-i\phi_{rt}}} E_{\text{in}}$$

Time-dependent equations



Interference condition at the input mirror

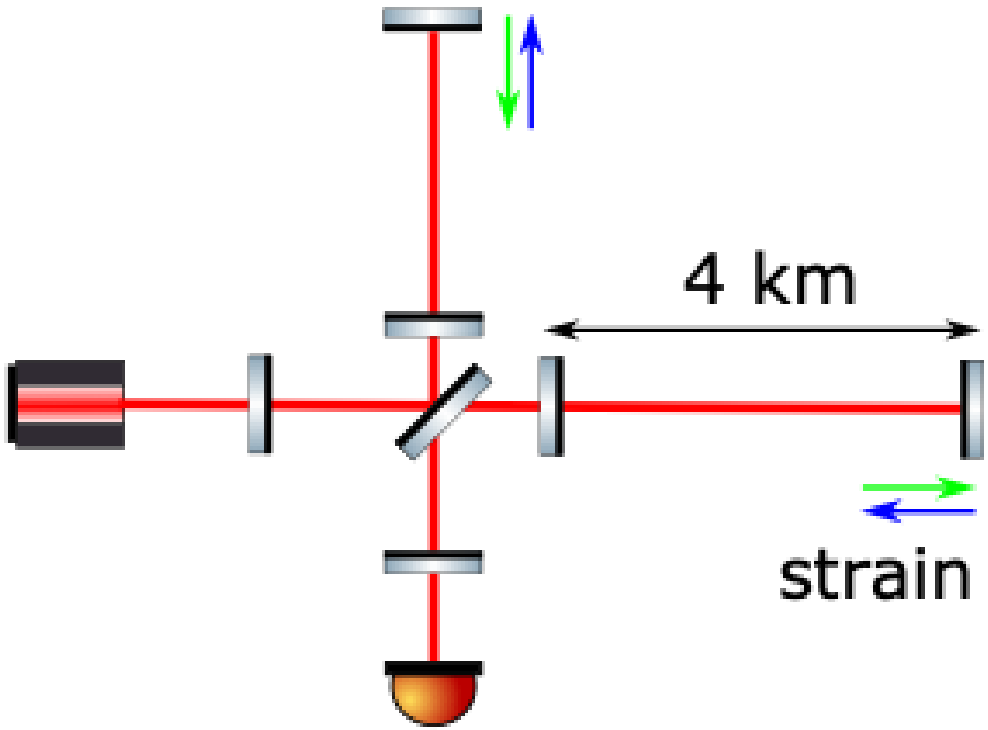
$$E_{\mathrm{cav}}(t) = E_{\mathrm{in}}\sqrt{T_1} + r_1 r_2 E_{\mathrm{cav}}(t-\tau)e^{-i\phi_{rt}}$$

We can derive the cavity pole from this equation (see the board)

Exercise 4

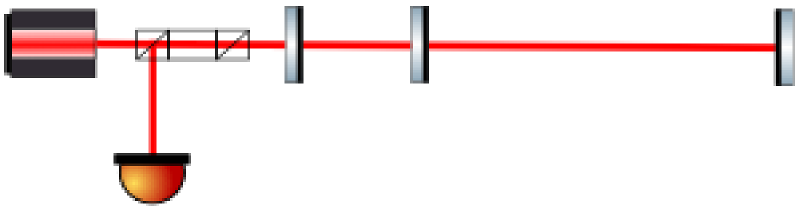
The LIGO arm is 3994.5 m long, and the power transmission of the input and output test masses is 1.48% and 5 ppm. Find the arm cavity pole.

Coupled cavities

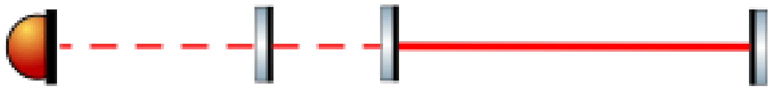


Equivalent LIGO layout

- Common arm: power pumping and laser stabilisation



- Differential arm: gravitational-wave detection

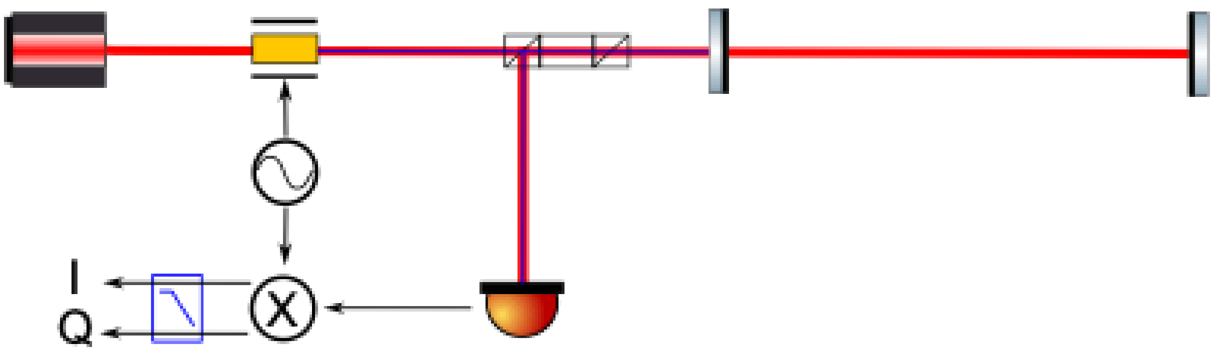


Exercise 4

Find the pole frequencies of the common and differential arm coupled cavities if power transmissivities of the power and signal recycling mirrors are 3% and 20%.

Pound-Drever-Hall (PDH) locking

- Can lock an interferometer on resonance.
- Need to take a derivative of the resonating power.
- Need to find fields that do not resonate and compare with the resonating field.



$$E_{out}^{eom} = E_{in}^{eom} e^{i\Gamma sin\Omega t} = E_{in}^{eom}(J_0(\Gamma) + \sum_{k=1}^{k=\infty} J_k(\Gamma)e^{ik\Omega t} + \sum_{k=1}^{k=\infty}(-1)^k J_k(\Gamma)e^{-ik\Omega t})$$

PDH locking

Reflected field from the cavity

$$E_{refl} = E_{in}(J_0(\Gamma)r(\omega_0) + J_1(\Gamma)r(\omega_0+\Omega)e^{i\Omega t} - J_1(\Gamma)r(\omega_0-\Omega)e^{-i\Omega t} + (2\Omega - terms))$$

$$P_{refl} = P_{refl}^{DC} + 2P_{in}J_0(\Gamma)J_1(\Gamma)(Im[S(\omega_0)]\sin\Omega t + Re[S(\omega_0)]\cos\Omega t) + (2\Omega - terms)$$

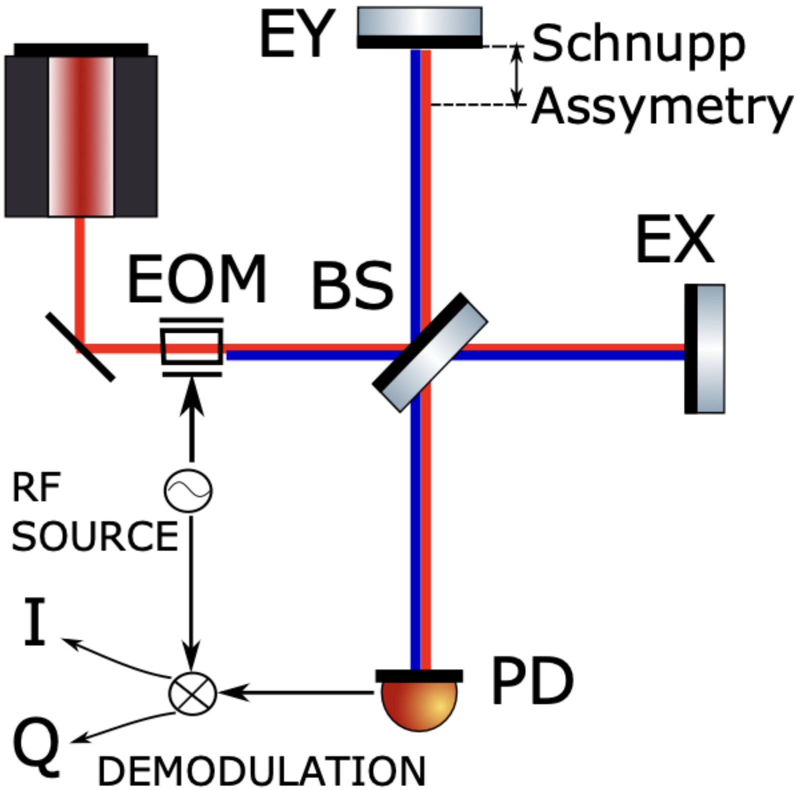
where $$S(\omega_0) = r(\omega_0)r(\omega_0+\Omega)^* - r(\omega_0)^* r(\omega_0-\Omega)$$

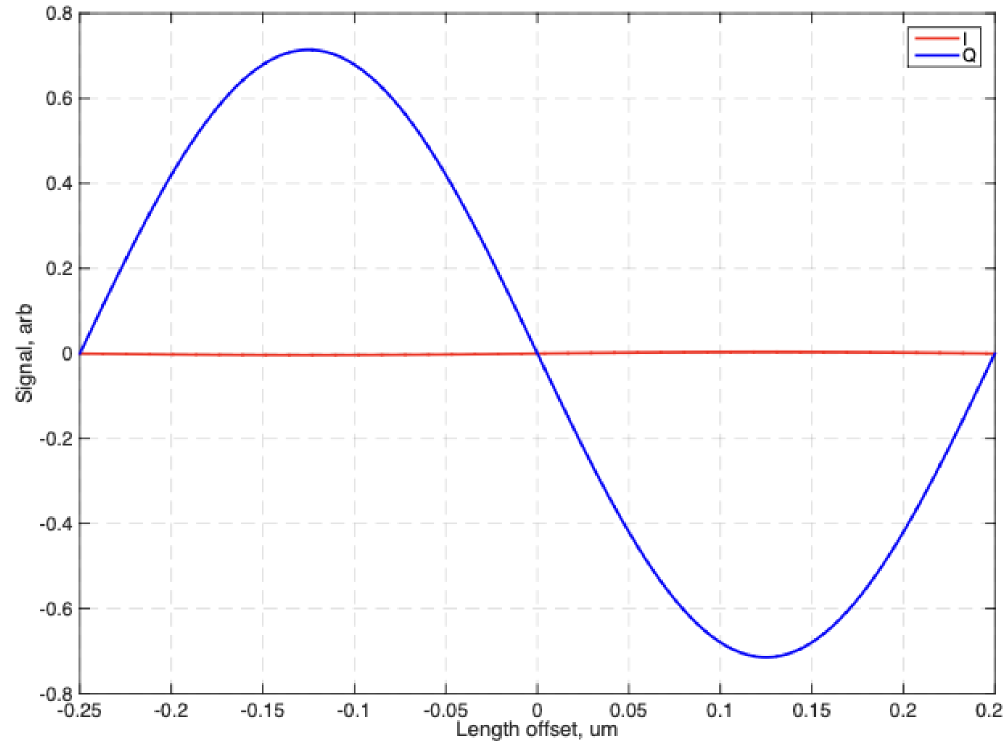
Near the cavity resonance

$$P_{refl}(t) = P_{refl}^{DC} - 16P_{in}J_0(\Gamma)J_1(\Gamma)\frac{F}{\lambda}x(t)\sin\Omega t$$

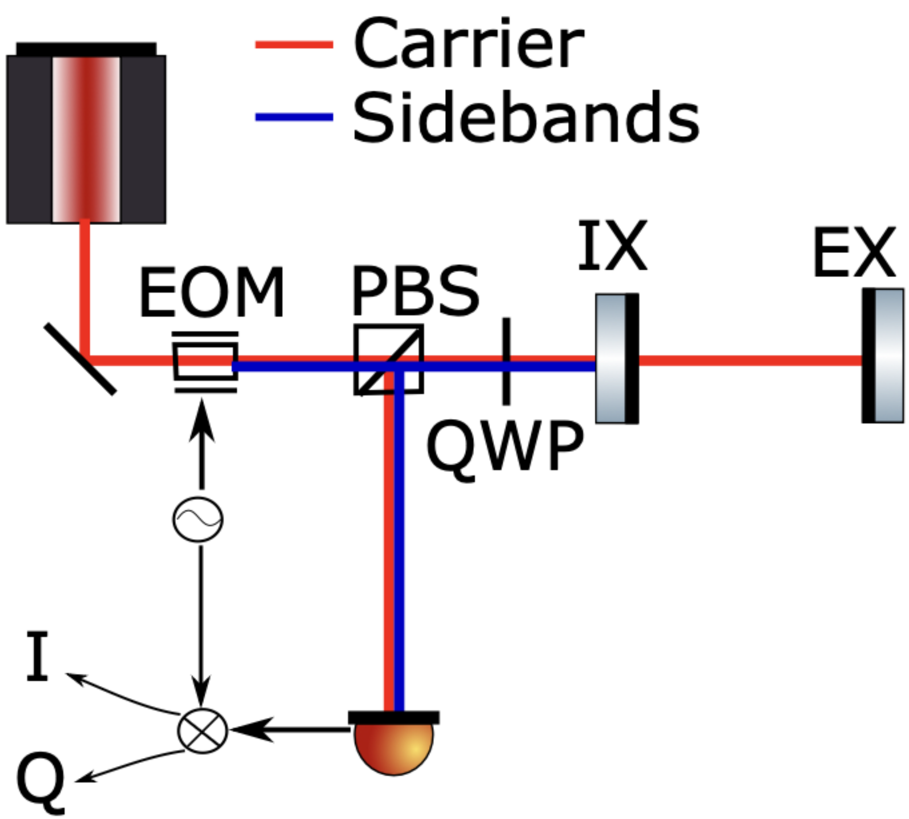
we get a linear response after demodulation.

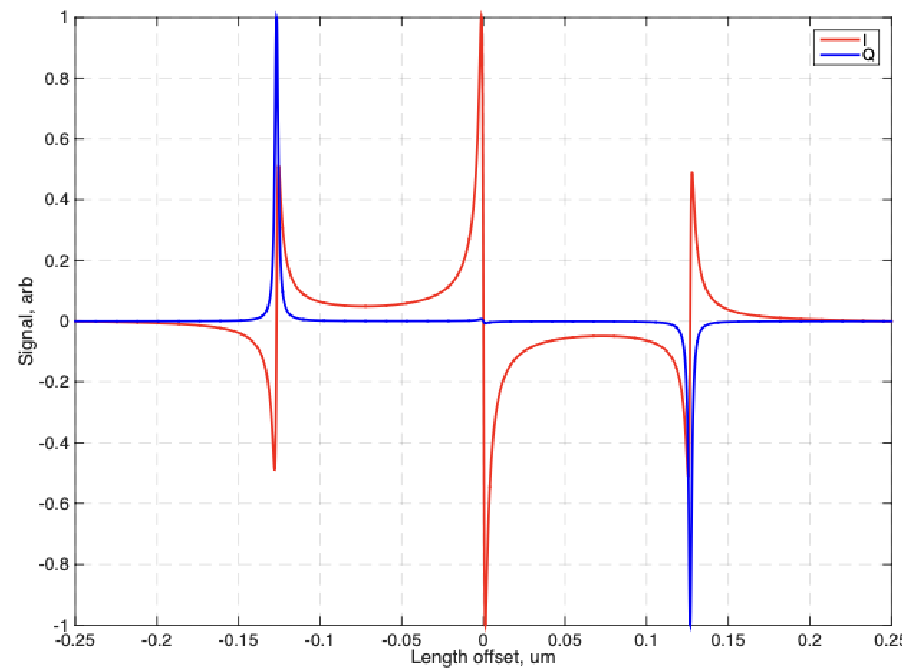
Example: Michelson interferometer





Example: Fabry-Perot cavity



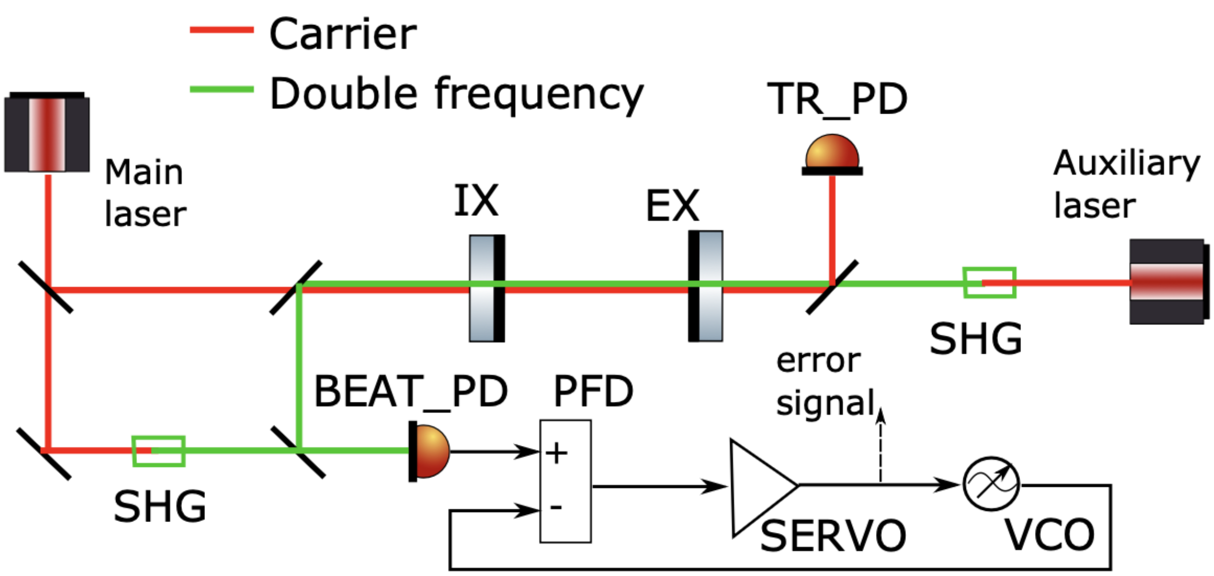


LIGO lock acquisition sequence

- 5 degrees of freedom are hard to catch at once
- Lock two arms with green and offset them from resonance
- Lock the power and signal recycling cavities and a Michelson interferometer at once (DRMI)
- Transition DRMI to 3-f signals
- Remove the CARM offset
- Transition DRMI back to 1-f signals and move DARM to DC readout

Arm length stabilisation (ALS)

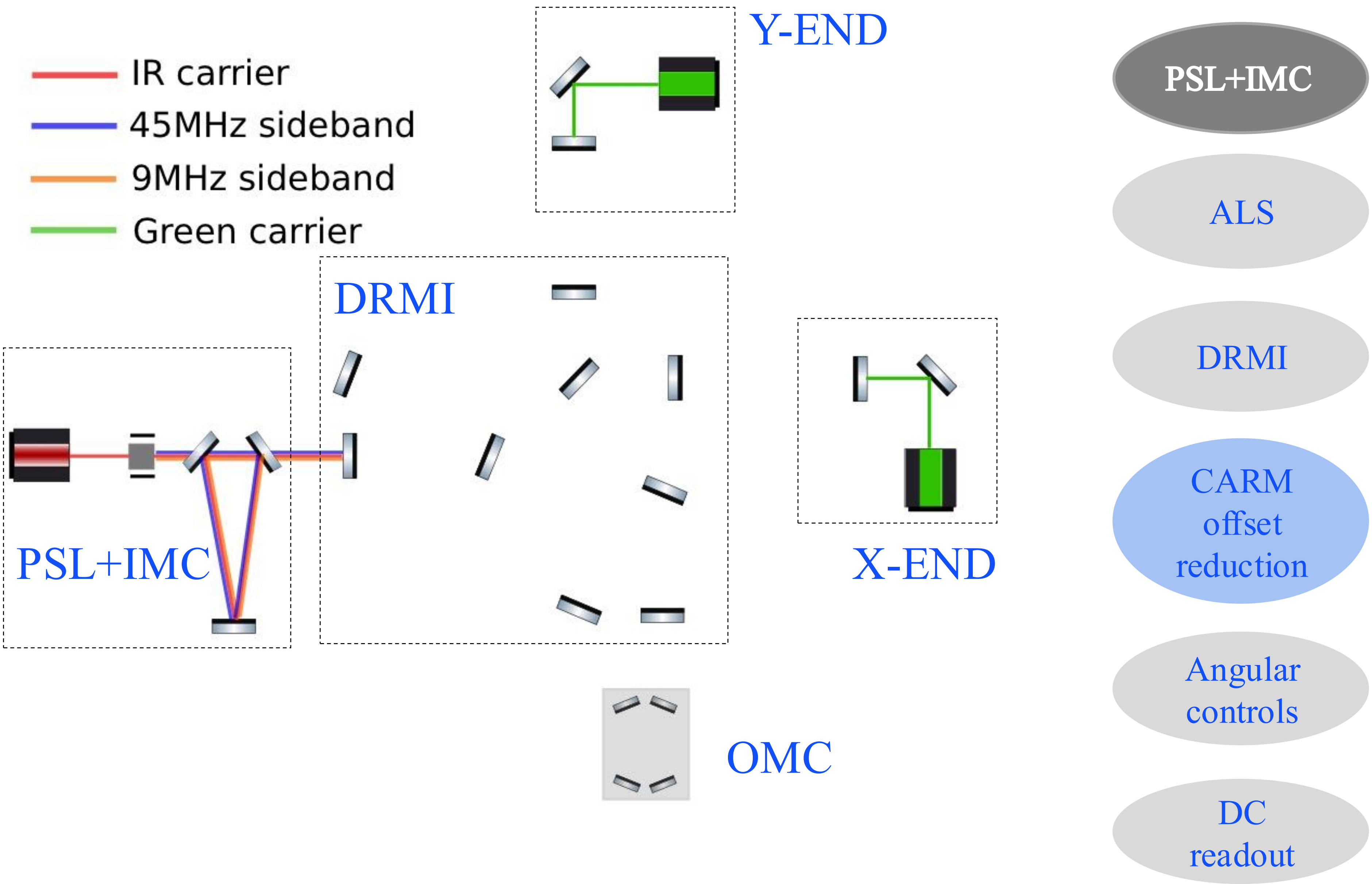
Use an auxiliary laser to control the arm cavities



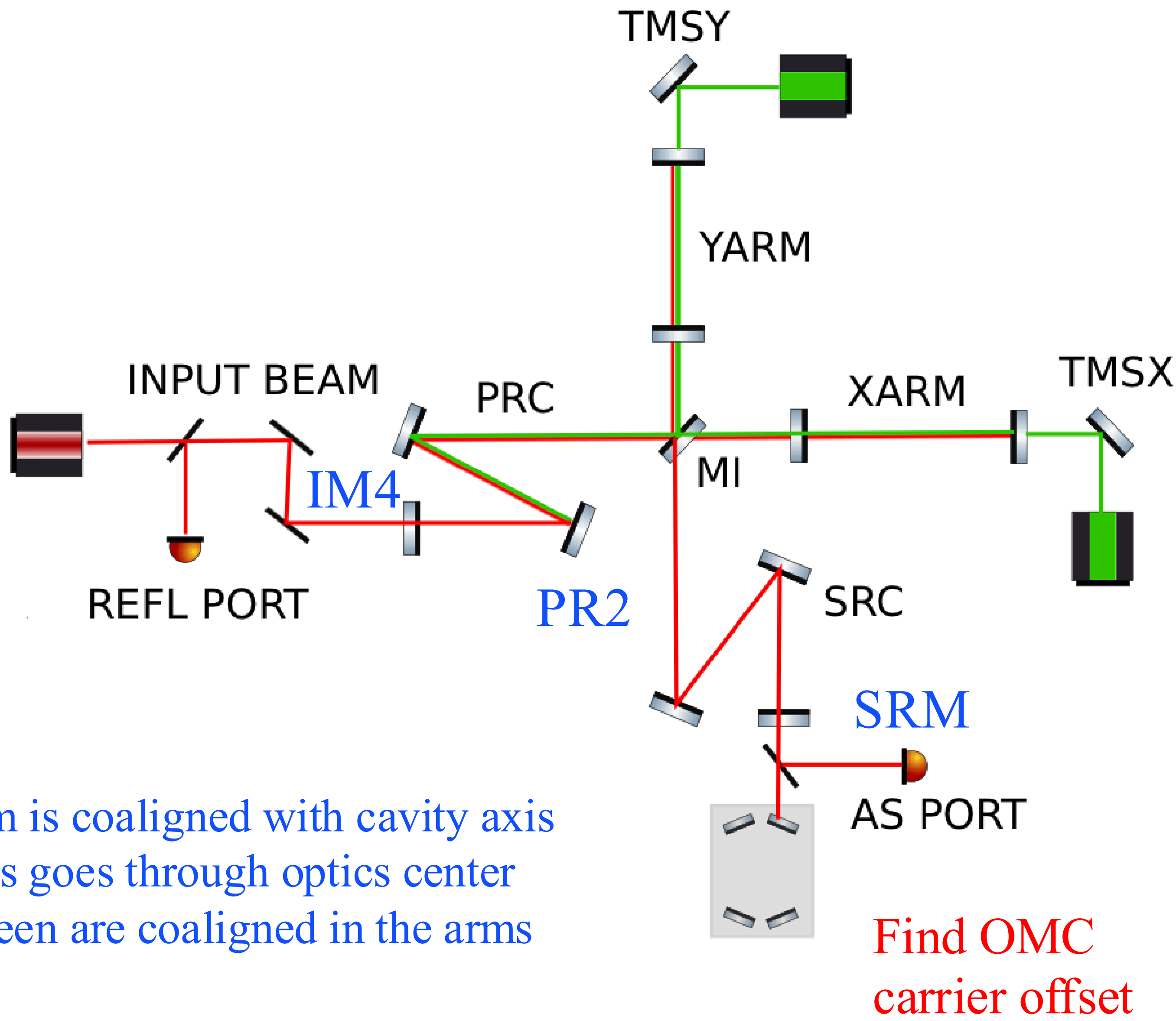
$$E_{beat} = E_l + E_{cav} = E_l^0 e^{i\omega_0 t - \varphi_0} + E_{cav}^0 e^{i\omega_{cav} - \varphi_{cav}}$$

$$P_{beat} = 2E_l^0 E_{cav}^0 \cos((\omega_0 - \omega_{cav})t - (\varphi_0 - \varphi_{cav})) + (DC - term)$$

Lock Acquisition Sequence

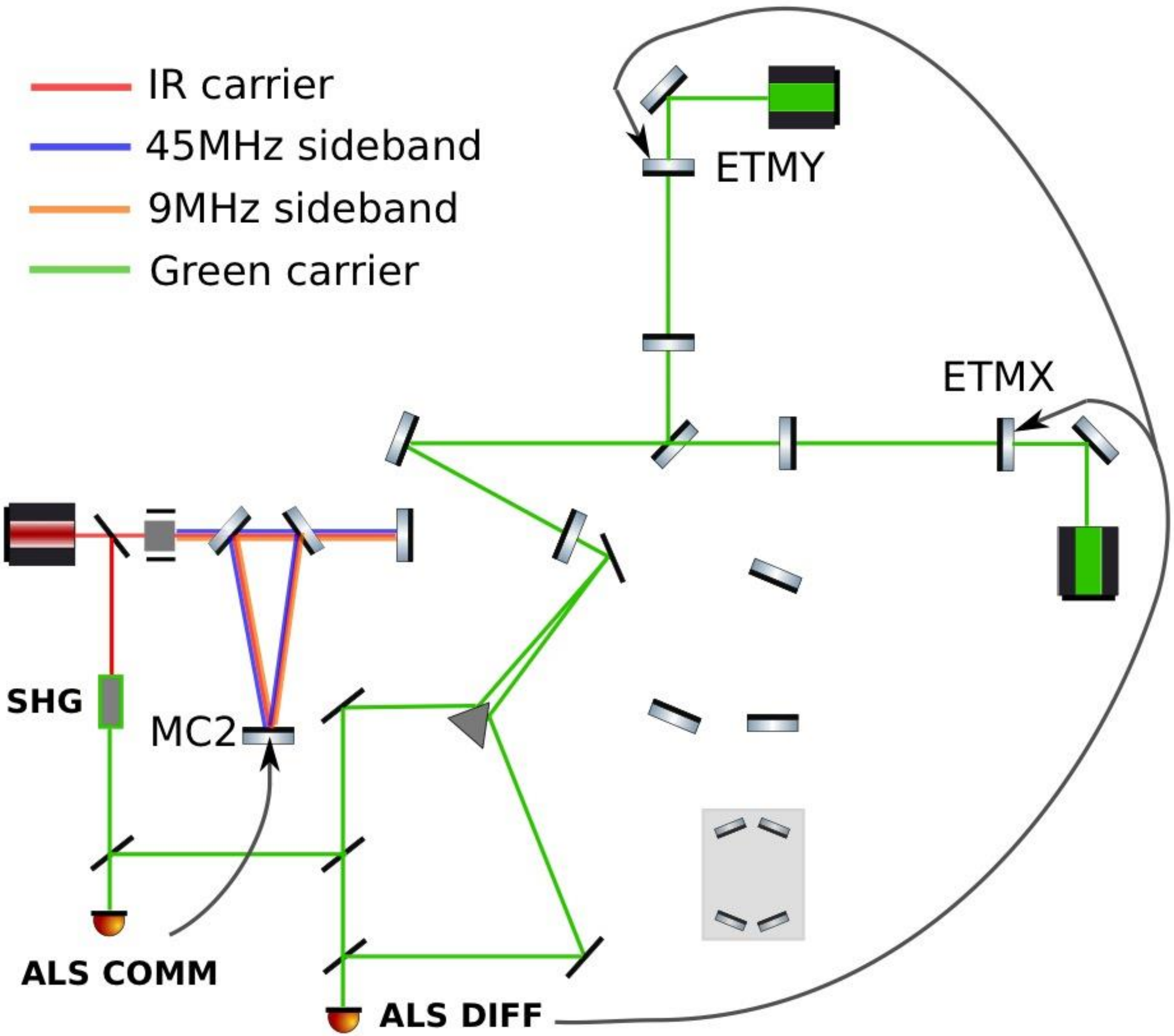


Initial alignment procedure



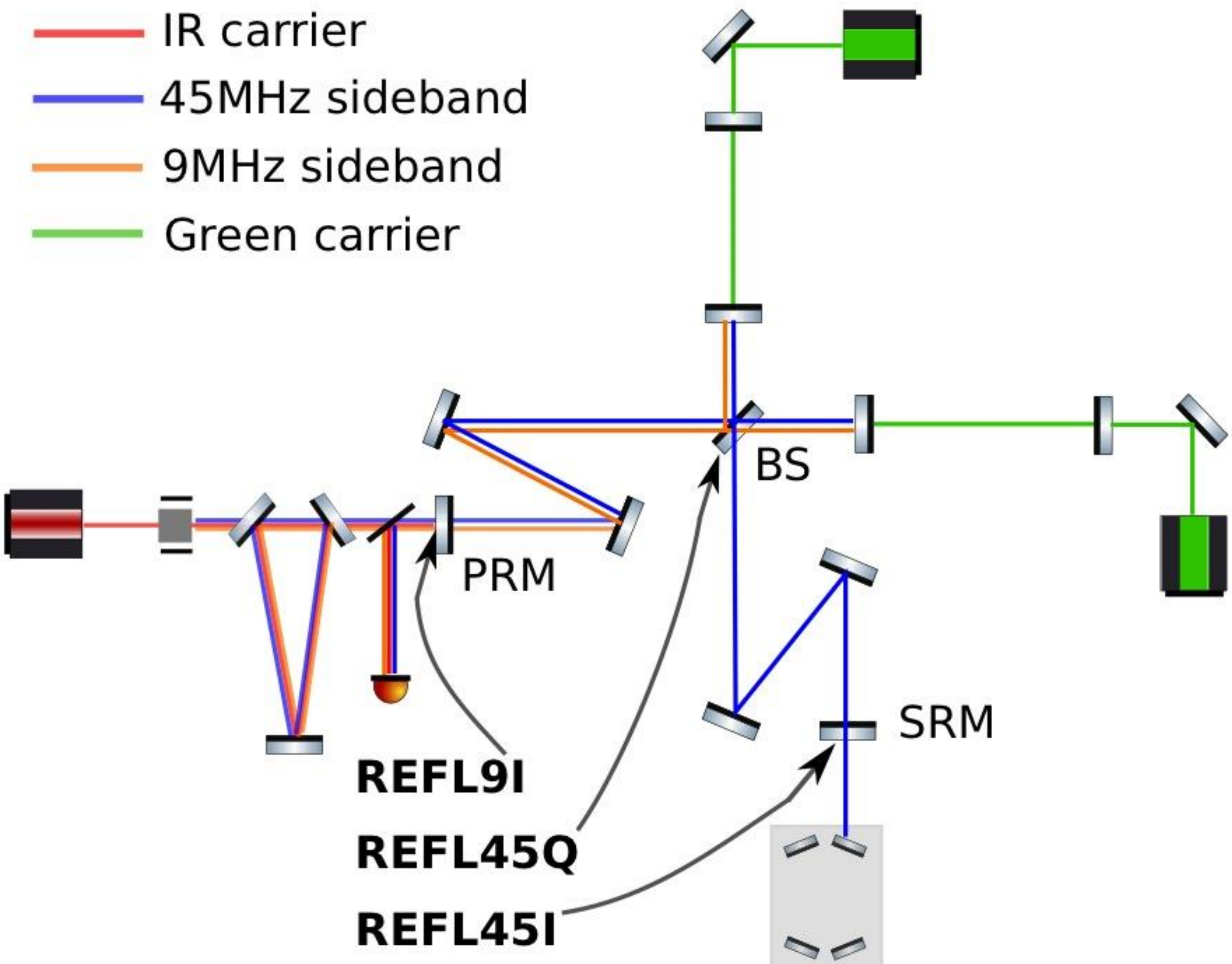
Goals:
- Input beam is coaligned with cavity axis
- Cavity axis goes through optics center
- IR and Green are coaligned in the arms

Lock Acquisition Sequence



- PSL+IMC
- ALS
- DRMI
- CARM offset reduction
- Angular controls
- DC readout

Lock Acquisition Sequence



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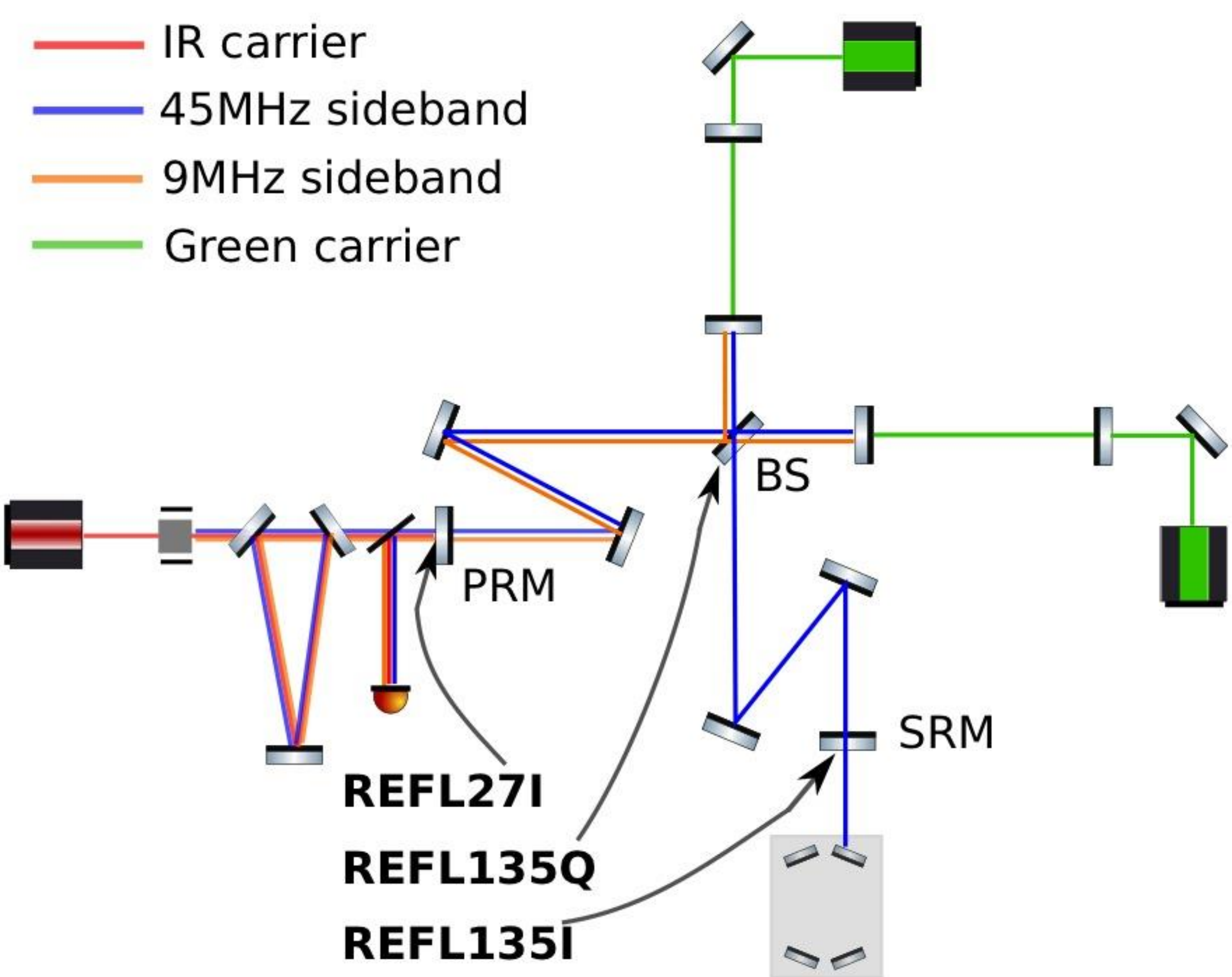
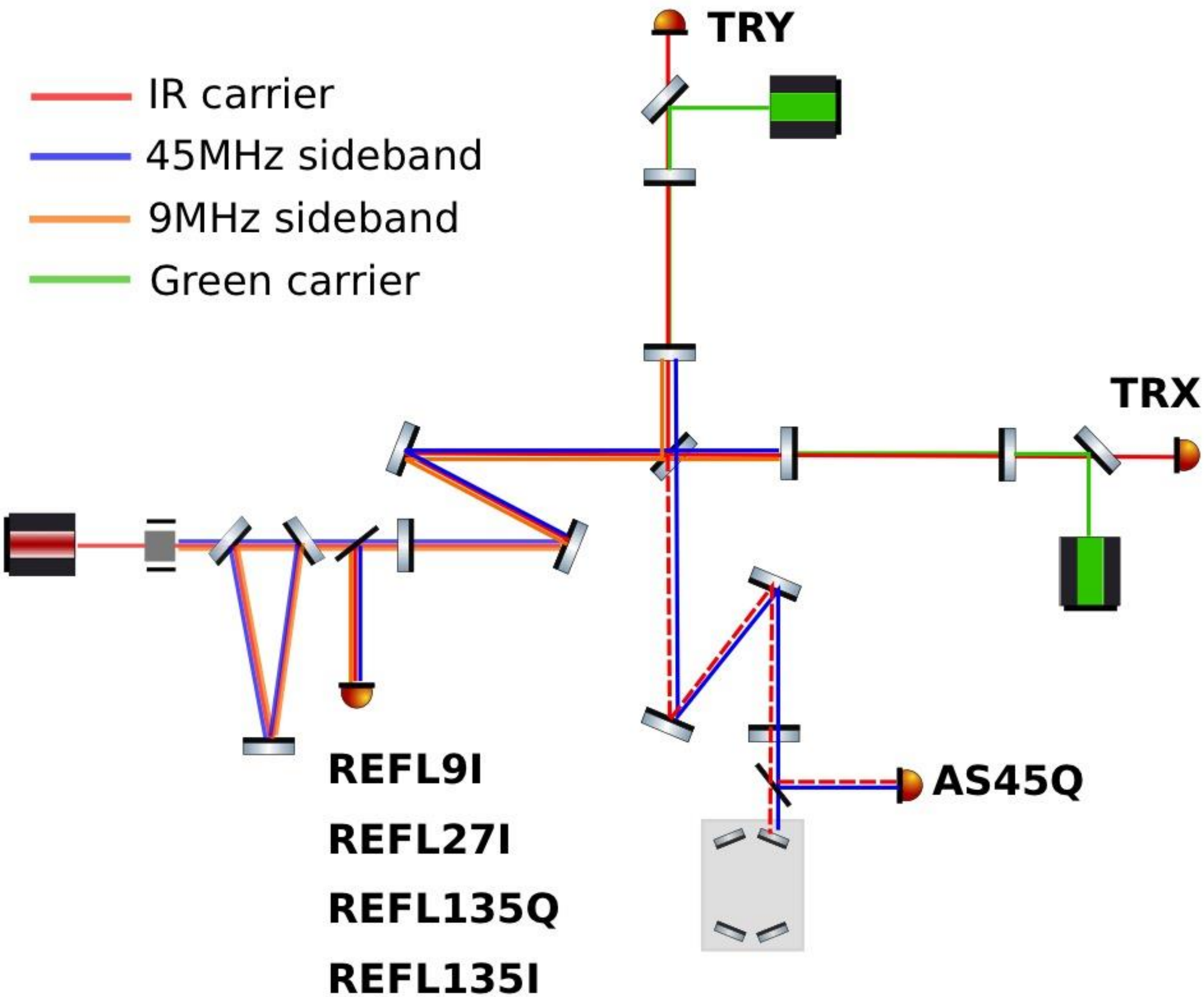


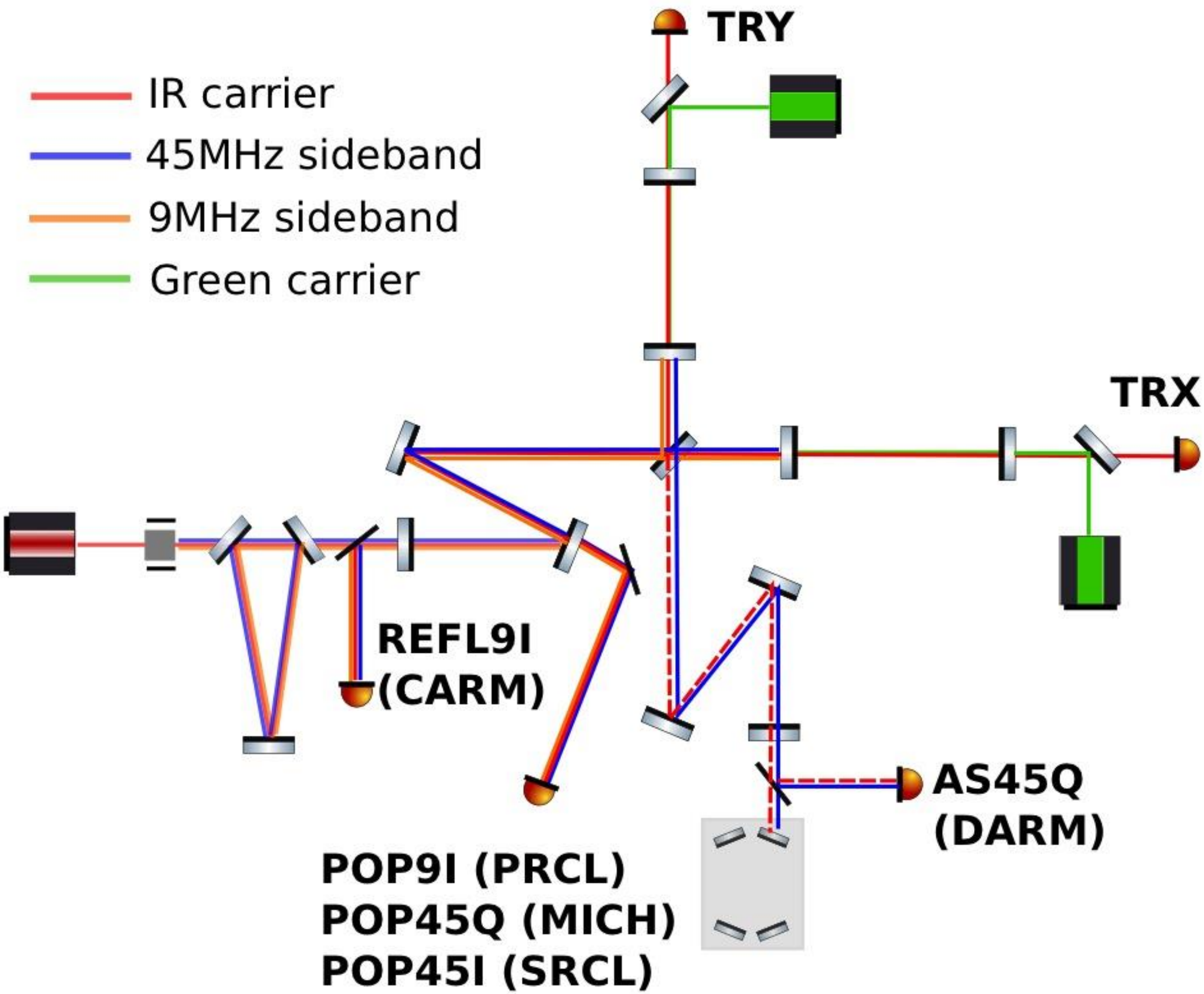
- PSL+IMC
- ALS
- DRMI
- CARM offset reduction
- Angular controls
- DC readout

Lock Acquisition Sequence



- PSL+IMC
- ALS
- DRMI
- **CARM offset reduction**
- Angular controls
- DC readout

Lock Acquisition Sequence



PSL+IMC

ALS

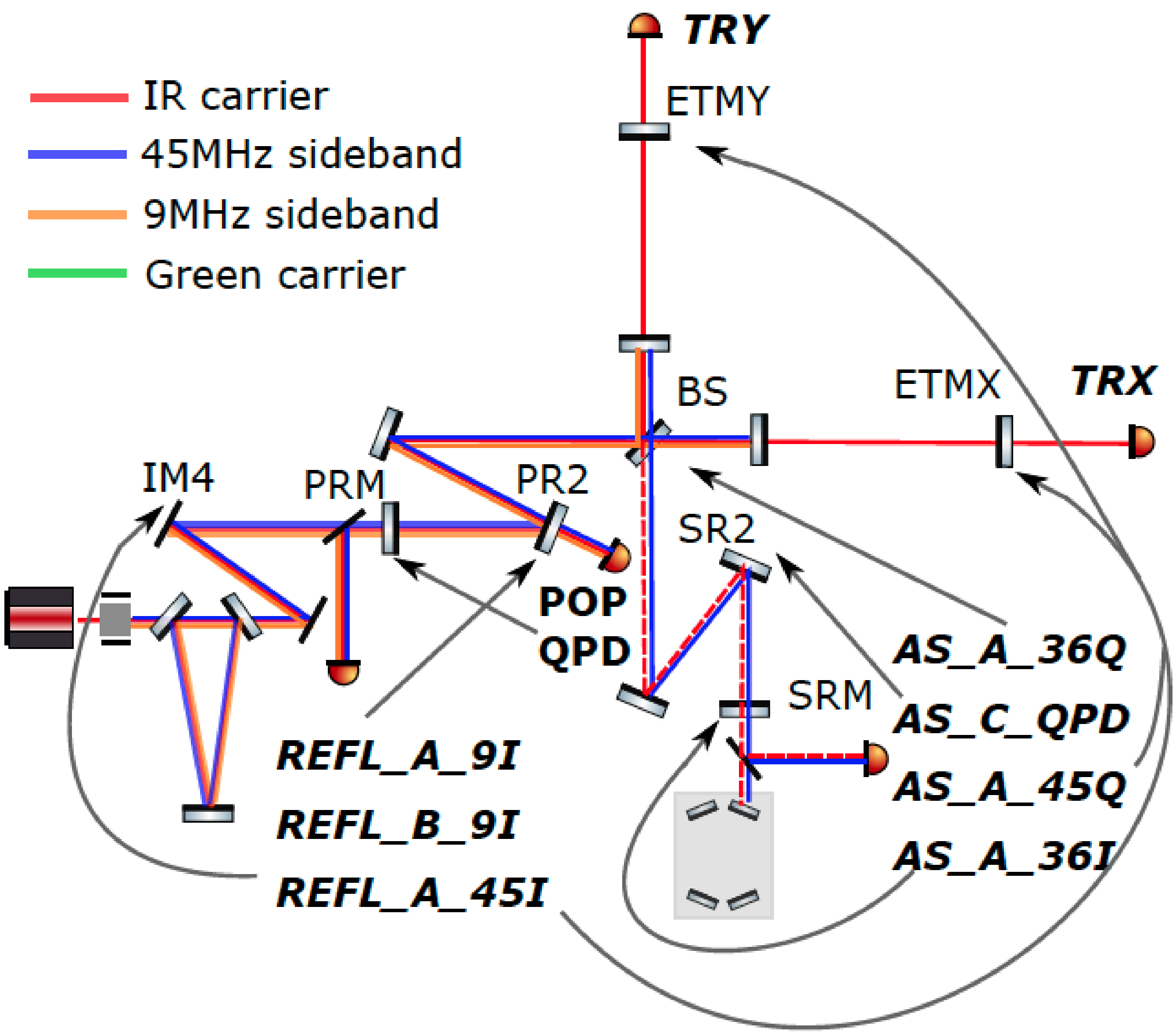
DRMI

CARM offset reduction

Angular controls

DC readout

Lock Acquisition Sequence



PSL+IMC

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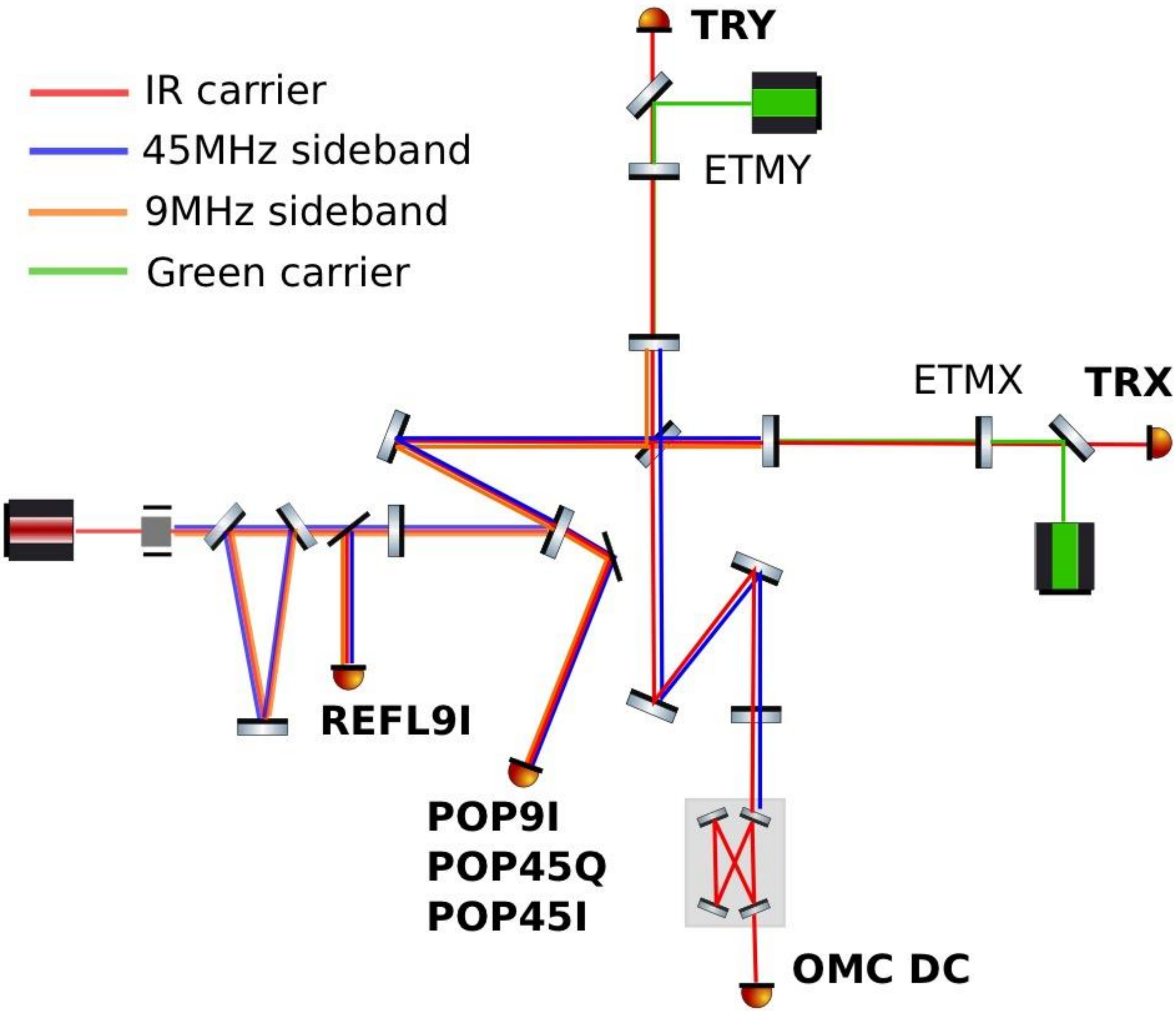
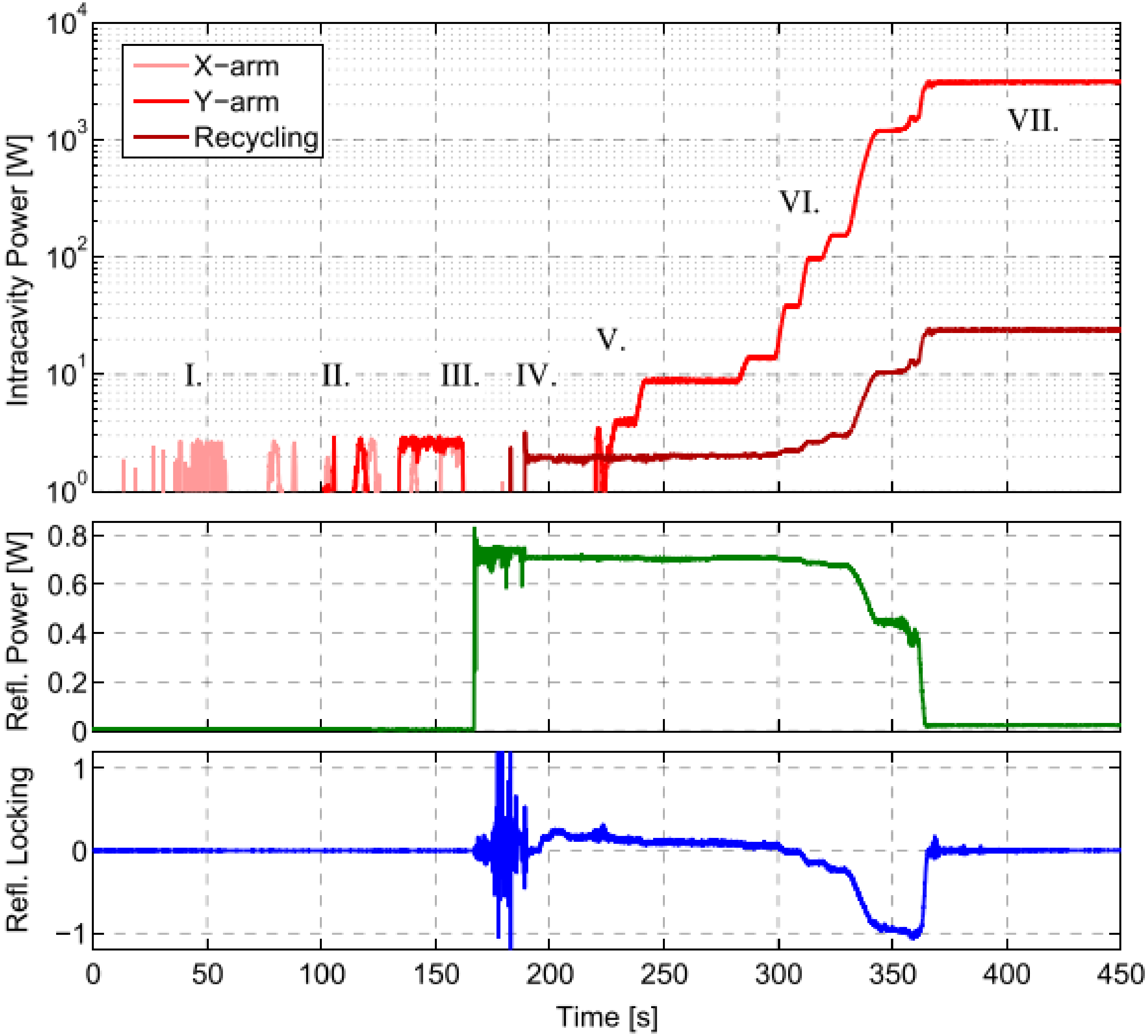
Lock Acquisition Sequence



- PSL+IMC
- ALS
- DRMI
- CARM offset reduction
- Angular controls
- DC readout


X−arm
Y−arm
Recycling
Intracavity Power [W]
10^4
10^3
10^2
10^1
10^0
I.
II.
III.
IV.
V.
VI.
VII.
Refl. Power [W]
0.8
0.6
0.4
0.2
0
Refl. Locking
1
0
−1
0
50
100
150
200
250
300
350
400
450
Time [s]

Exercise 5

Simulate the transfer functions from all 5 longitudinal degrees of freedom to the sensors in reflection and antisymmetric ports.