IUCAA workshop
Day 2: HAM control

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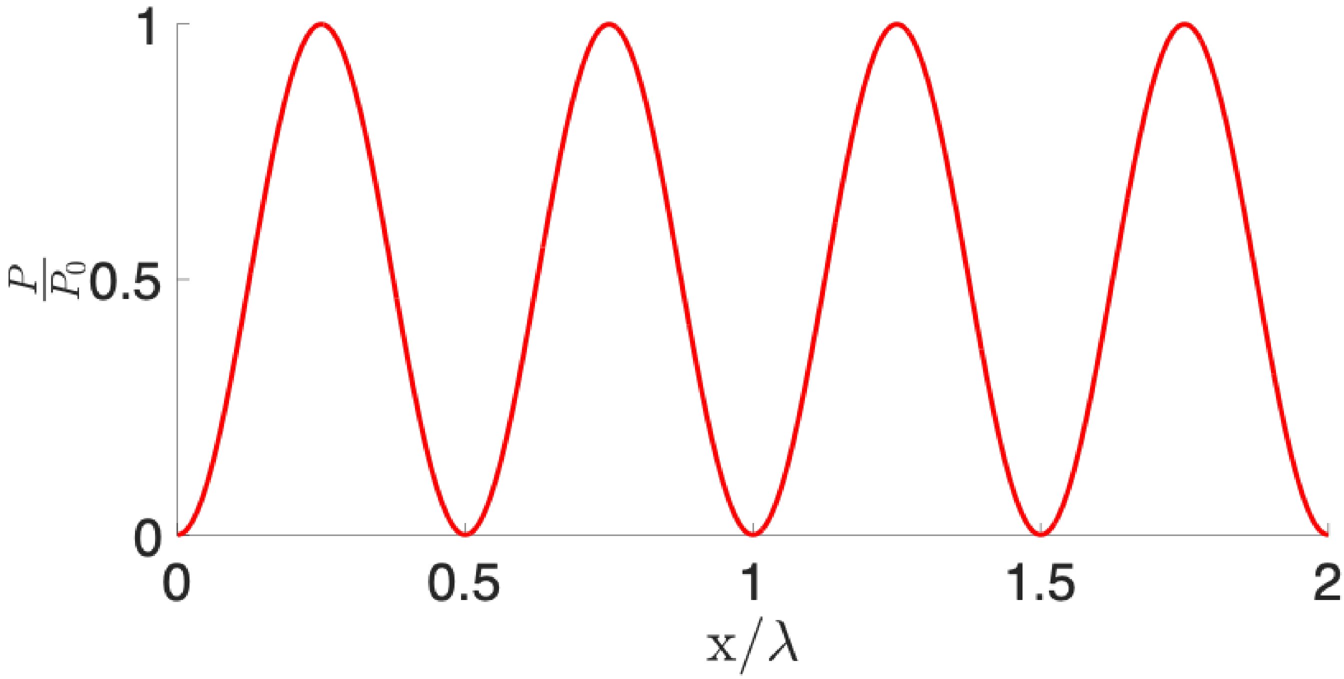
The programme

- Day 1: Interferometry and feedback control
- Day 2: Mechatronics
- Day 3: Cavity locking
- Day 4: Angular controls and noises
- Day 5: Practical issues and solutions

Recap: Michelson interferometer

For convenience, we set $L_x - L_y = N * \text{lambda} + x$.



The power swings from min to max if the target mirror moves by 266 nm.

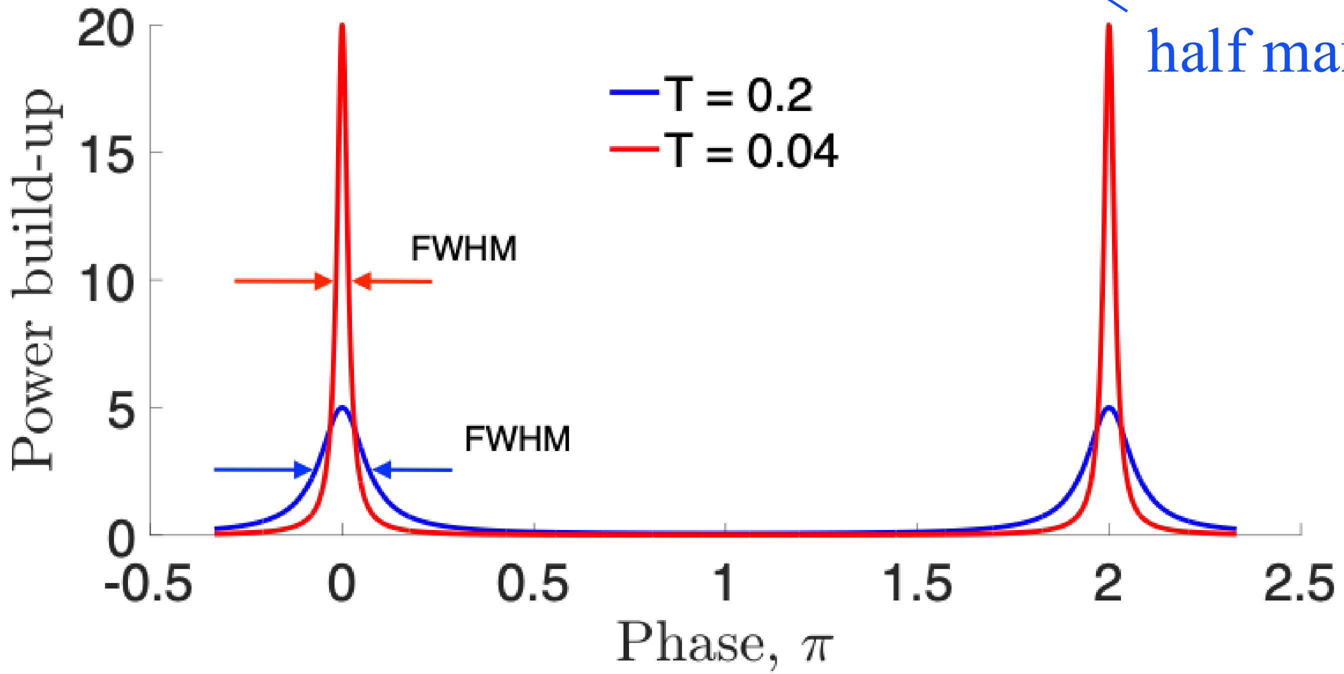
Recap: Fabry-Perot interferometer

- Intracavity power $$P_{\text{cav}} = \frac{T}{|1 - r^2 e^{-i\phi_{rt}}|^2} P_{\text{in}}.$$

$T = t^2$
$R = r^2$

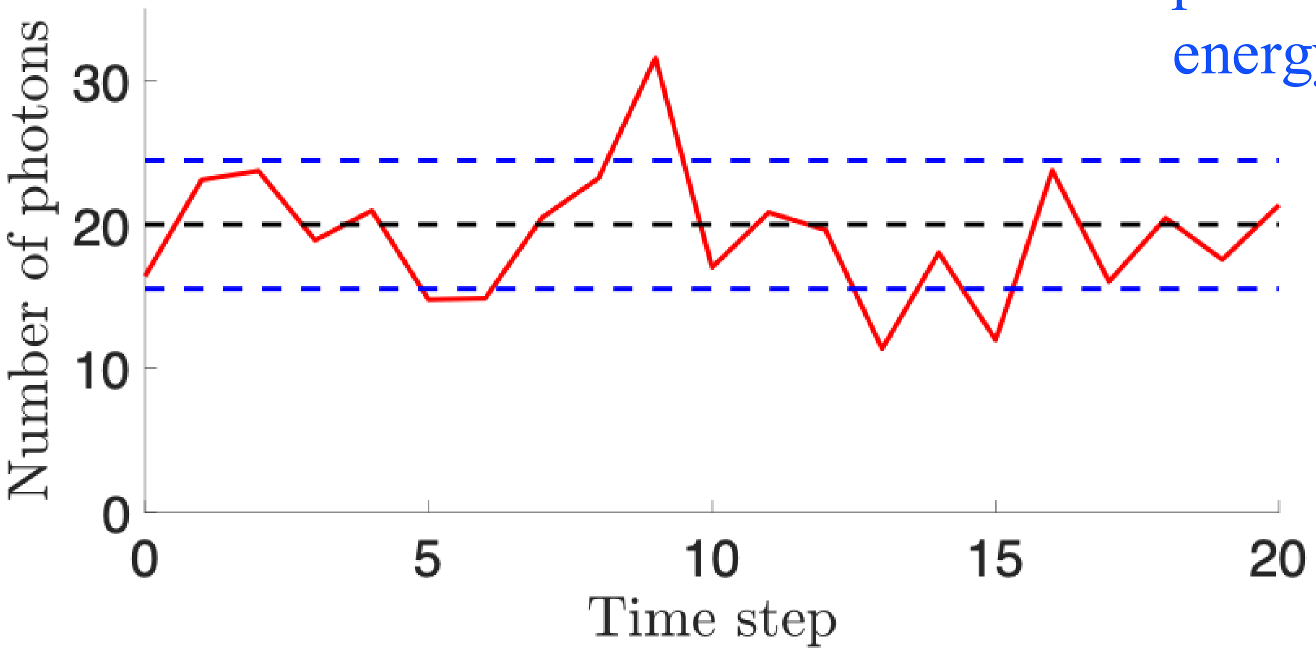
- Define the optical finesse as $$\mathcal{F} = \frac{2\pi}{\Delta\phi_{rt}}$$

Full width at half max (FWHM)



Recap: Shot noise

- The power fluctuates even if the sample mirror does not move.
- The average number of photons $\langle N \rangle = \frac{P\tau}{h\nu}$, τ ← measurement time; $h\nu$ ← photon energy
- The standard deviation is $\sqrt{\langle N \rangle}$.

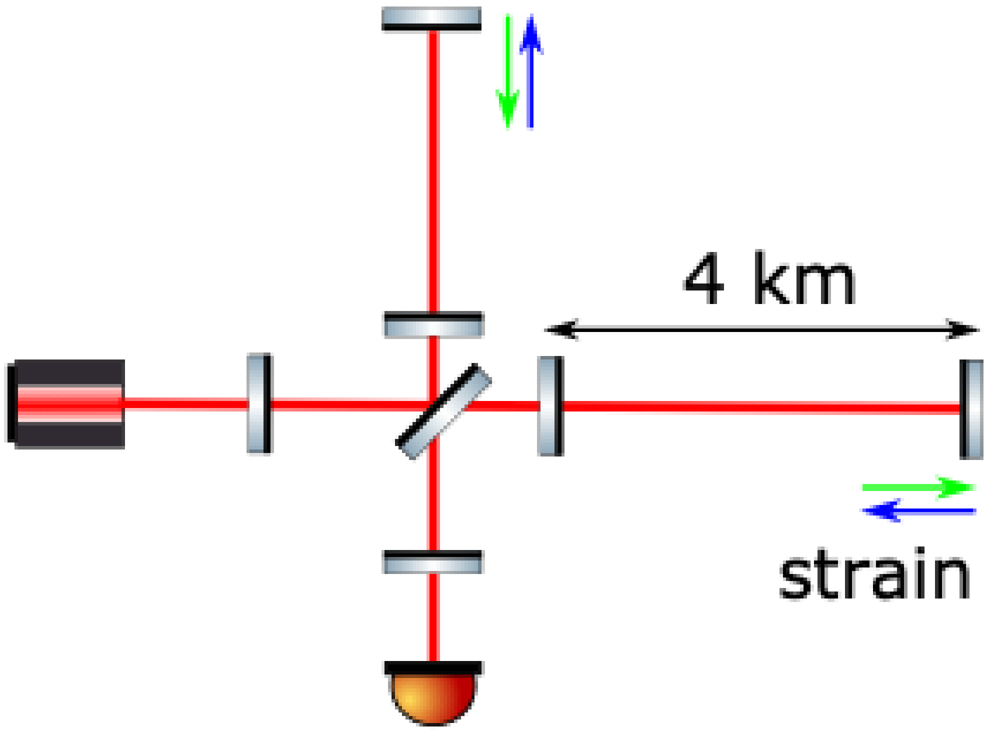


Summary of noises

- Sensing noises
  - Shot noise
  - Electronics noise
  - Laser frequency noise
  - ….
- Displacement noises
  - Seismic noise
  - Thermal noises
  - Gravity gradients
  - Quantum back action noise
  - …
- A dedicated discussion on Day 4.

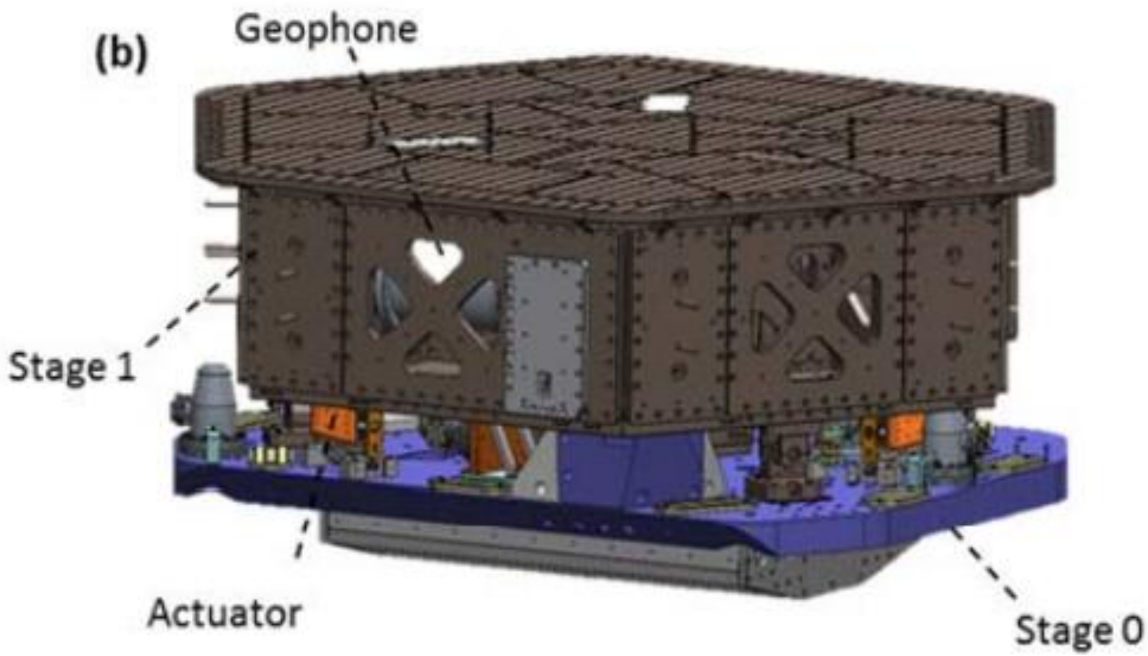
Seismic noise

Ground -> active platform -> test mass.



Internal seismic isolation (ISI)

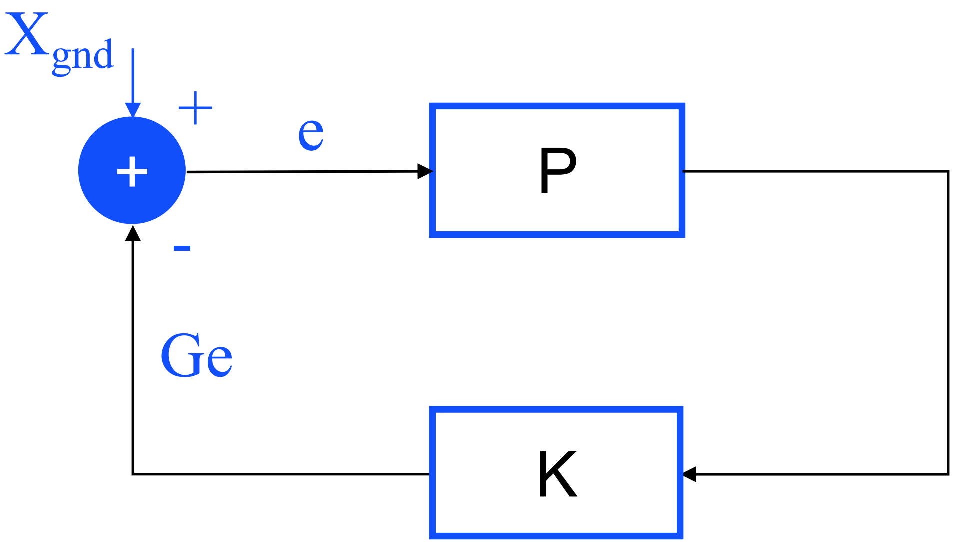
- A suspended optical table with sensors and coil-magnet actuators.
- Test masses are suspended from these platforms.



Overview

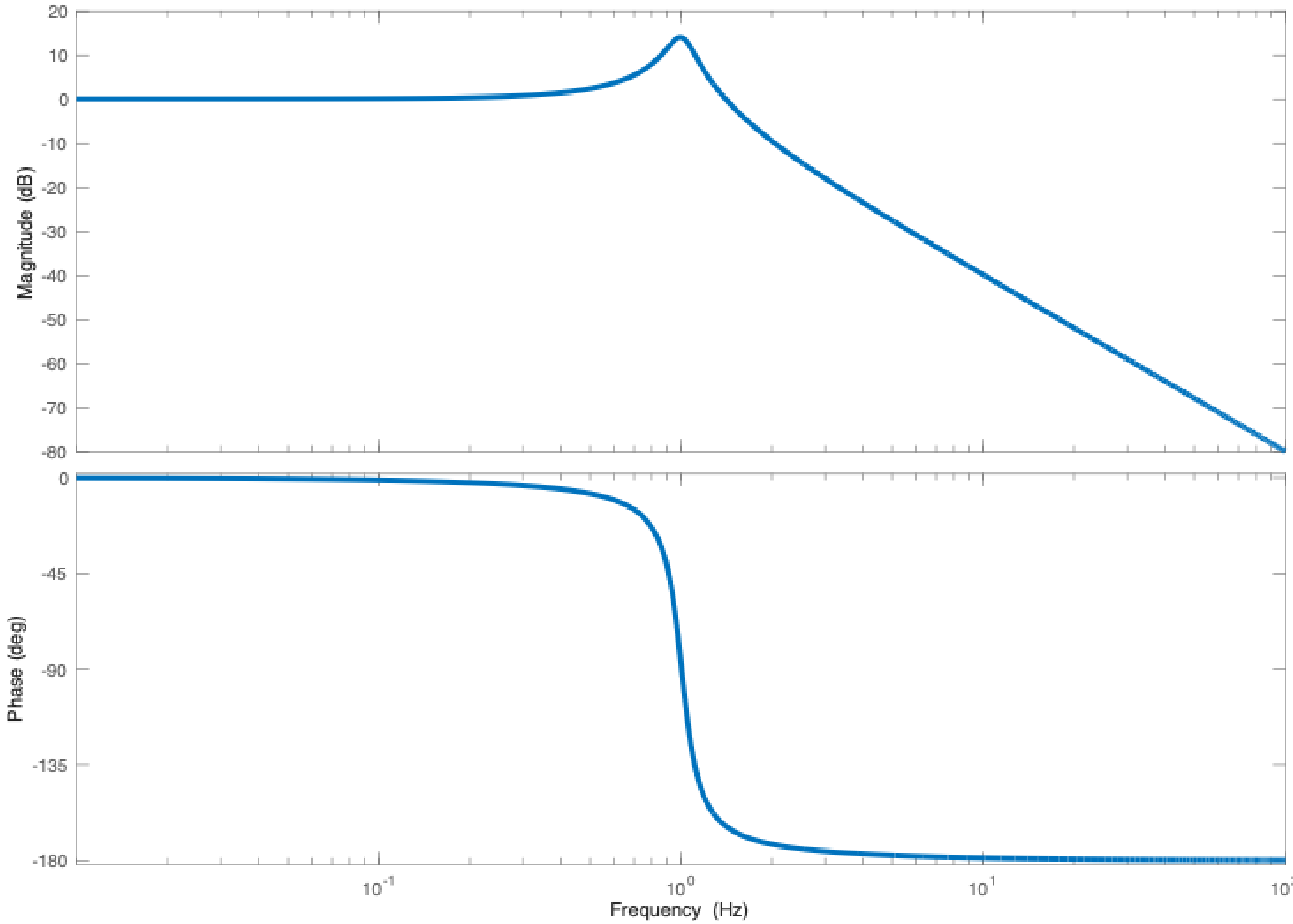
- Stability of the ISI feedback control loop
- Seismometer frequency response
- Sensor blending
- Sensor correction
- Practical limitations

Simplified feedback control of the ISI



- Need to suppress the sensor signal.
- Define G = PK is the open loop transfer function.
- Then CL = 1 / (1 + G) is the closed loop transfer function.
- CL must be stable!
- $e = CL\ X_{gnd}$

ISI response, P



Exercise 1.

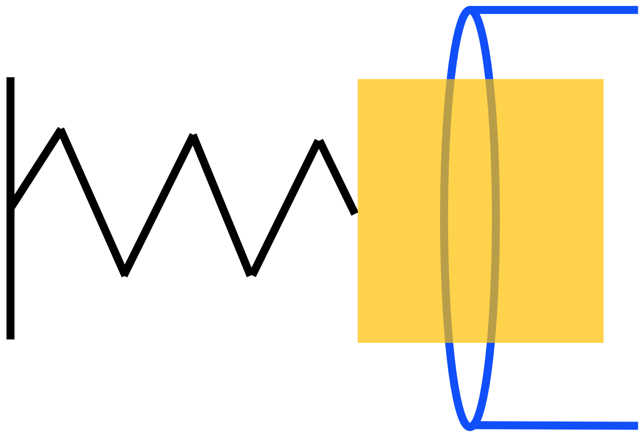
a) Consider $G = C / s^n$, where n is an integer. Determine n and the sign of C for which $CL = 1 / (1 + G)$ is a stable function. Plot the frequency response of a stable CL?

b) Design a feedback controller K which suppresses the ISI motion by a factor of 100 at 1 Hz. Make sure that the closed loop of the system is stable.

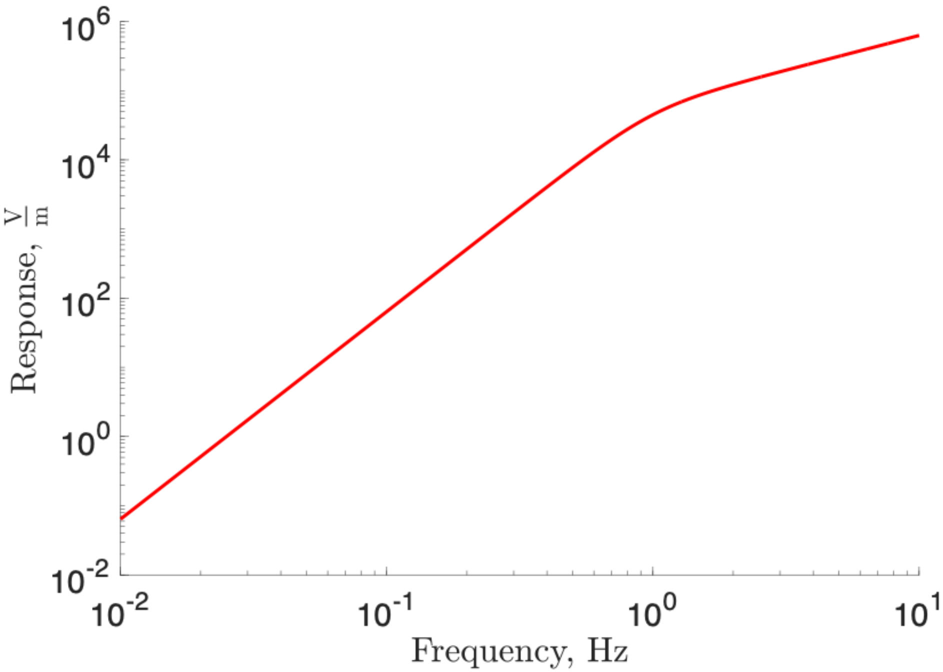
Assume $P = 40 / (s^2 + s + 40)$.

Inertial sensors: response

- Reduces at low frequencies.
- Consist of a mass on a spring with a relative readout.



Magnitude of the response:



Equations of motion:

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = \omega_0^2 x_{\mathrm{gnd}} + \gamma\dot{x}_{\mathrm{gnd}}; \quad x_m = \dot{x}_{\mathrm{gnd}} - \dot{x}$$

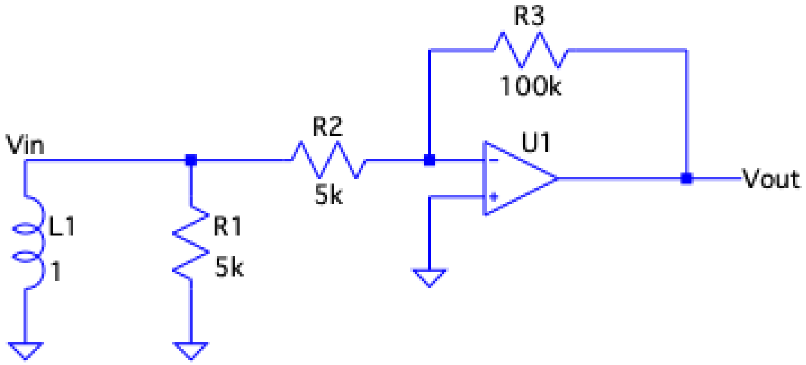
$$s^2 x + \gamma s x + \omega_0^2 x = \omega_0^2 x_{\mathrm{gnd}} + s\gamma x_{\mathrm{gnd}}; \quad x_m = s(x_{\mathrm{gnd}} - x)$$

Measurement: $$x_m = \frac{s^3}{s^2 + \gamma s + \omega_0^2} x_{\mathrm{gnd}}$$

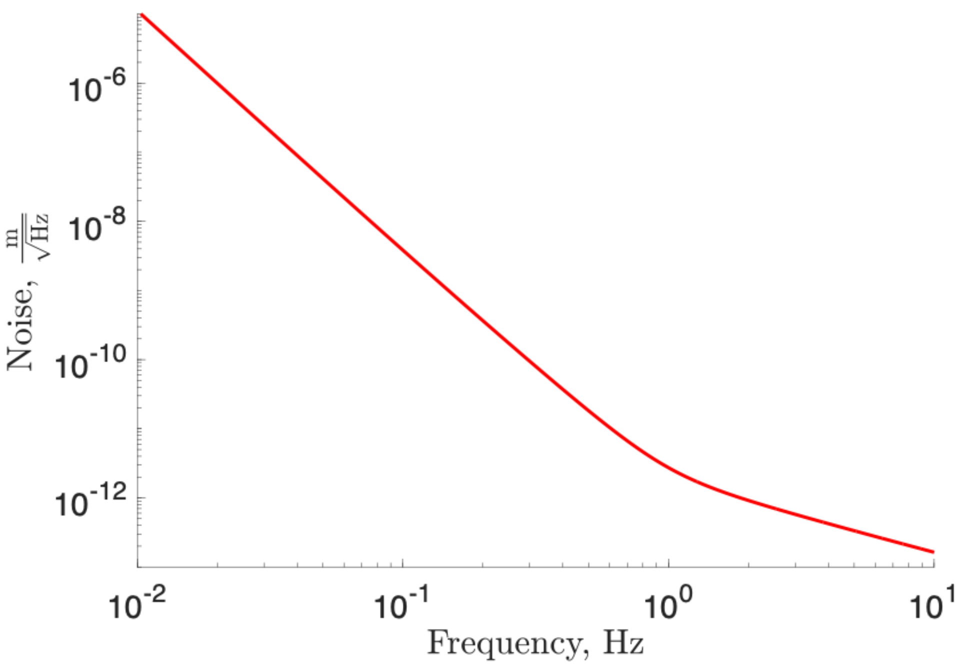
Inertial sensors: noise

- Increases at low frequencies
- Thermal and readout

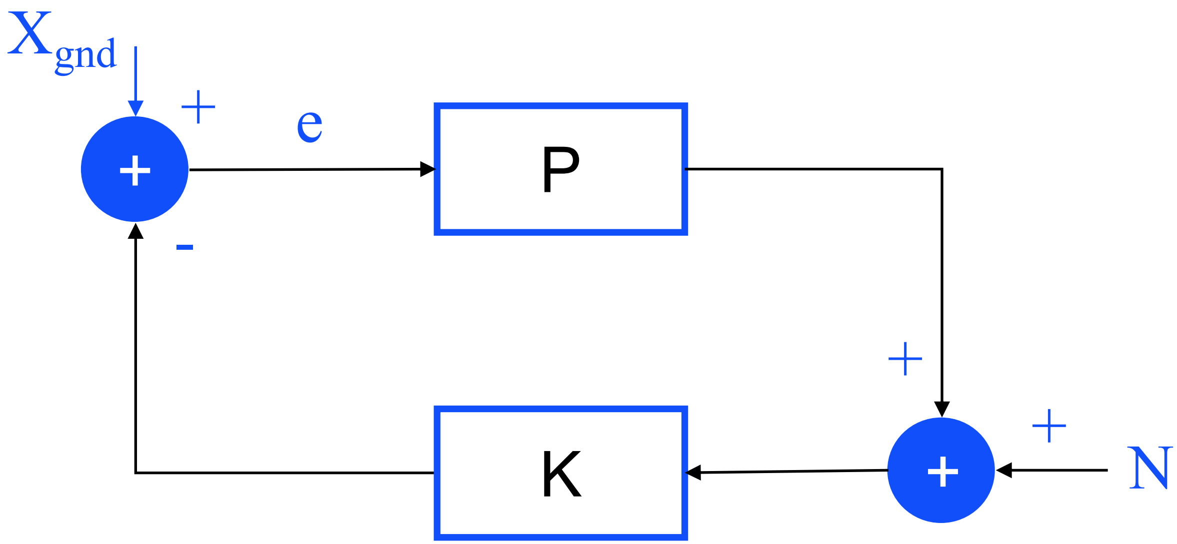
Simplified readout scheme:



Calibrated noise:



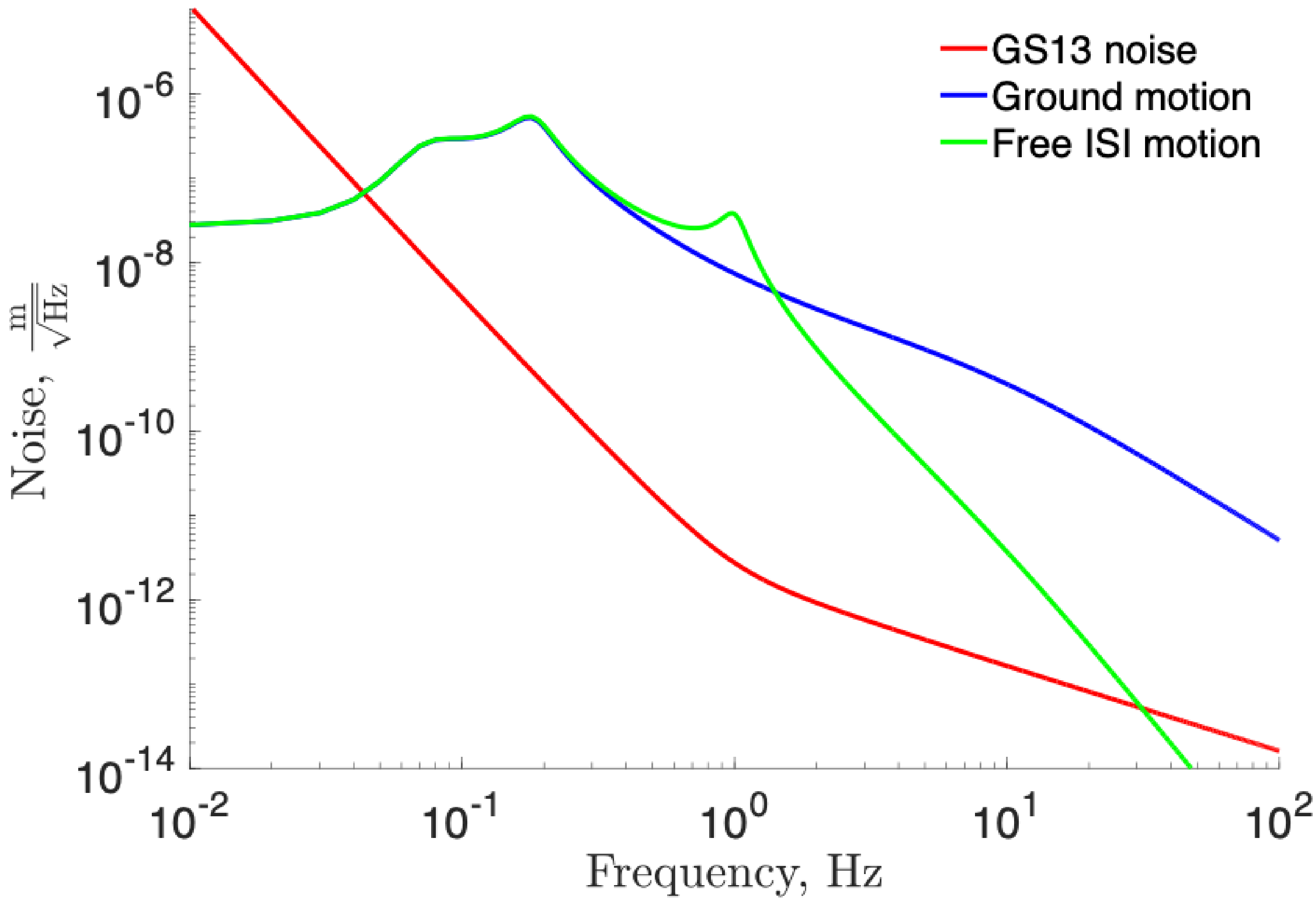
ISI control with sensor noise



The platform motion follows the sensor noise N (see the board).

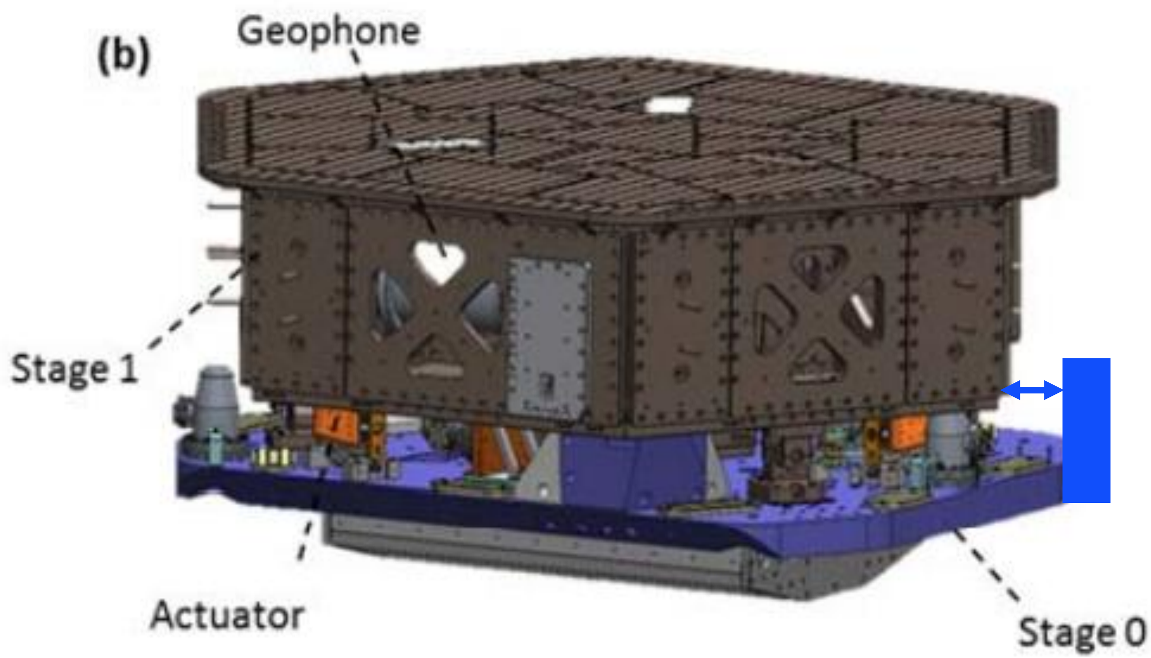
Useful question: how does the system behave in the high gain regime when G >> 1?

ISI free swinging motion



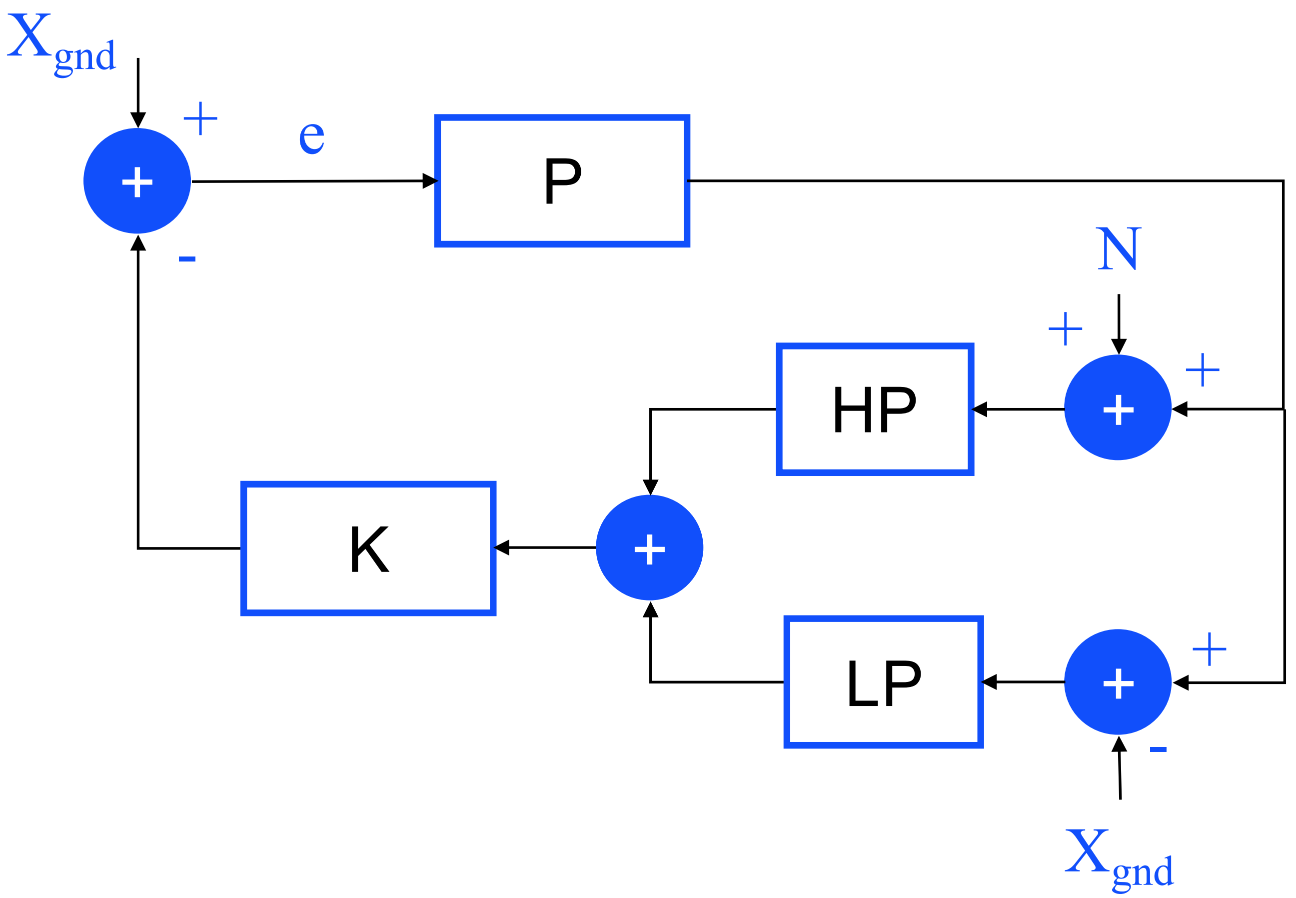
“Put” the ISI on the ground

- Measure the distance between the ISI and the ground
- Bring the signal in the ISI control loop

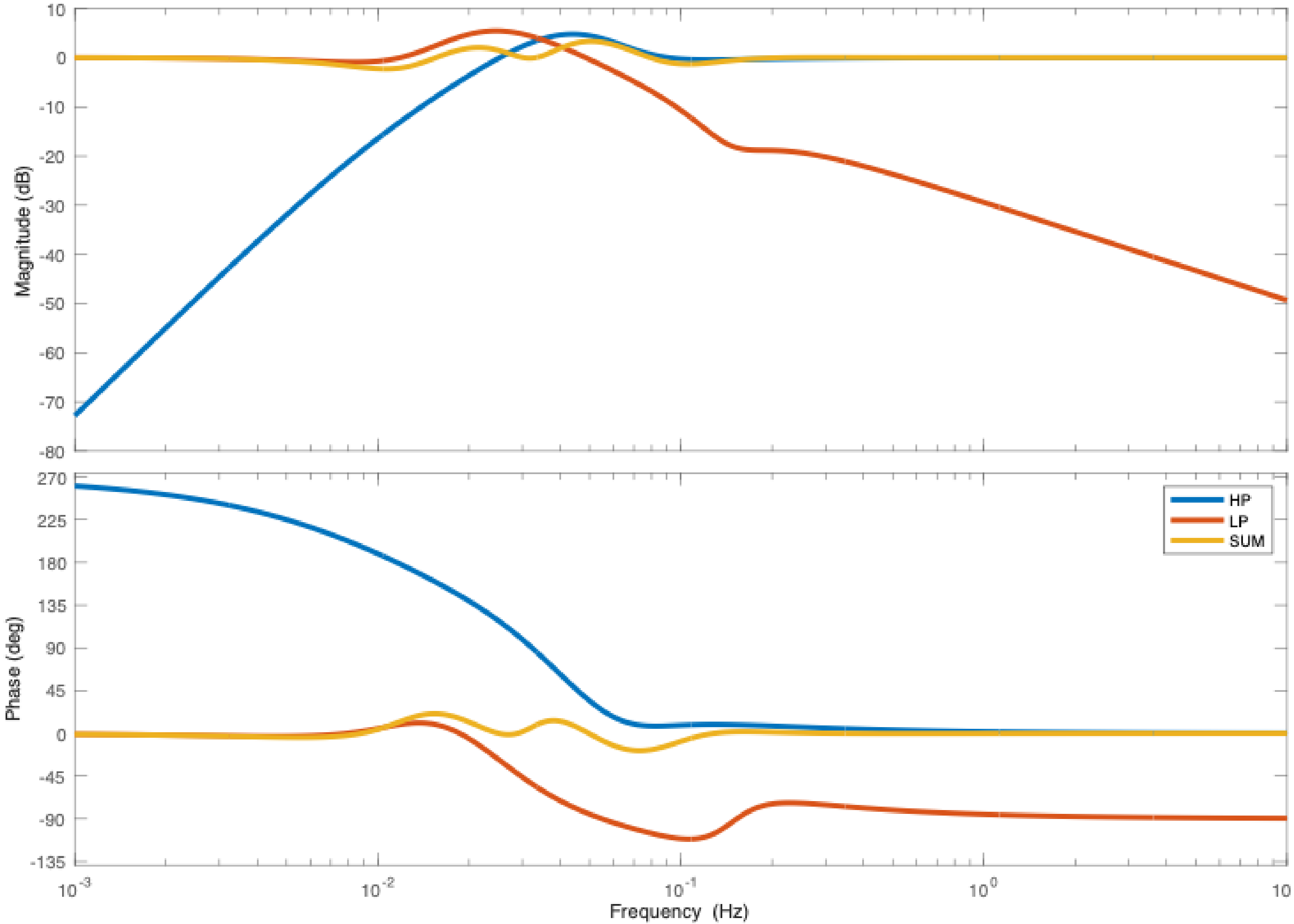


Sensor blending

- Measure the distance between the ISI and the ground.

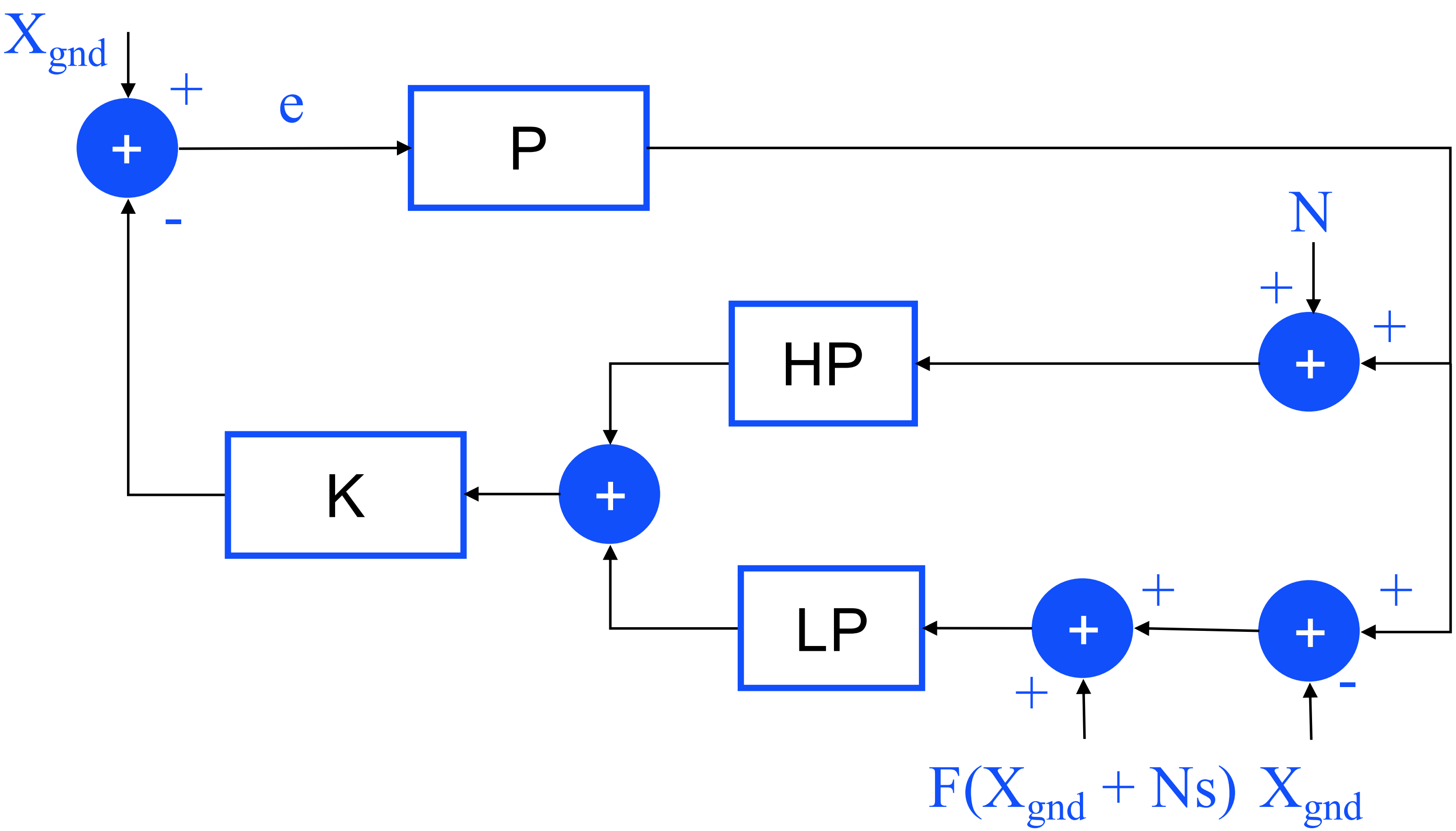


Example of blending filters



Sensor correction

Measure the ground motion with a seismometer and correct the displacement sensor signal.



Practical limitations

- Noises from various sensors
- Ground motion changes over time
- Blends must be stable
- The ISI isolation loop must be stable
- Mechanical resonances of the ISI

Exercise 2

Design blending filters, sensor correction filters, and find the ISI motion. Plot the spectra of noises that contribute to the motion.